

PARI-GP Reference Card

(PARI-GP version 2.5.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: **gp**
to exit GP, type **\q** or **quit**

Help

describe function **?function**
extended description **??keyword**
list of relevant help topics **???pattern**

Input/Output & Defaults

output previous line, the lines before **%, %', %'', etc.**
output from line n **%n**
separate multiple statements on line **;**
extend statement on additional lines ****
extend statements on several lines **{seq₁; seq₂};**
comment **/* ... */**
one-line comment, rest of line ignored **\\ ...**
set default d to val **default({d}, {val}, {flag})**
mimic behavior of GP 1.39 **default(compatible,3)**

Metacommands

toggle timer on/off **#**
print time for last result **##**
print % n in raw format **\a n**
print defaults **\d**
set debug level to n **\g n**
set memory debug level to n **\gm n**
enable/disable logfile **\l {filename}**
print % n in pretty matrix format **\m**
set output mode (raw=0, default=1) **\o n**
set n significant digits **\p n**
set n terms in series **\ps n**
quit GP **\q**
print the list of PARI types **\t**
print the list of user-defined functions **\u**
read file into GP **\r filename**
write % n to file **\w n filename**

GP Within Emacs

to enter GP from within Emacs: **M-x gp, C-u M-x gp**
word completion **(TAB)**
help menu window **M-\c**
describe function **M-?**
display T_EX'd PARI manual **M-x gpman**
set prompt string **M-\p**
break line at column 100, insert **M-**
PARI metacommand \textit{letter} **M-\letter**

Reserved Variable Names

$\pi = 3.14159\dots$ **Pi**
Euler's constant $= .57721\dots$ **Euler**
square root of -1 **I**
big-oh notation **O**

PARI Types & Input Formats

t_INT/t_REAL. Integers, Reals $\pm n, \pm n.ddd$
t_INTMOD. Integers modulo m **Mod(n, m)**
t_FRAC. Rational Numbers n/m
t_FFELT. Elt in a Finite Field **ffgen(T)**
t_COMPLEX. Complex Numbers $x + y * I$
t_PADIC. p -adic Numbers $x + O(p^k)$
t_QUAD. Quadratic Numbers $x + y * \text{quadgen}(D)$
t_POLMOD. Polynomials modulo g **Mod(f, g)**
t_POL. Polynomials $a * x^n + \dots + b$
t_SER. Power Series $f + O(x^k)$
t_QFI/t_QFR. Imag/Real bin. quad. forms **Qfb($a, b, c, \{d\}$)**
t_RFRAC. Rational Functions f/g
t_VEC/t_COL. Row/Column Vectors $[x, y, z], [x, y, z]~$
t_MAT. Matrices $[x, y, z; t; u, v]$
t_LIST. Lists **List($[x, y, z]$)**
t_STR. Strings **"aaa"**

Standard Operators

basic operations $+, -, *, /, ^$
i=i+1, i=i-1, i=i*j, ... **i++, i--, i*=j, ...**
euclidean quotient, remainder $x \backslash y, x \backslash y, x \% y, \text{divrem}(x, y)$
shift x left or right n bits $x << n, x >> n$ or **shift($x, \pm n$)**
comparison operators $<=, <, >=, >, ==, !=$
boolean operators (or, and, not) **||, &&, !**
sign of $x = -1, 0, 1$ **sign(x)**
maximum/minimum of x and y **max, min(x, y)**
integer or real factorial of x **x! or factorial(x)**
derivative of f w.r.t. x **f'**

Conversions

Change Objects

to vector, matrix, set, list, string **Col/Vec, Mat, Set, List, Str**
create PARI object ($x \bmod y$) **Mod(x, y)**
make x a polynomial of v **Pol($x, \{v\}$)**
as above, starting with constant term **Polrev($x, \{v\}$)**
make x a power series of v **Ser($x, \{v\}$)**
PARI type of object x **type(x)**
object x with precision n **prec($x, \{n\}$)**
evaluate f replacing vars by their value **eval(f)**

Select Pieces of an Object

length of x **#x or length(x)**
 n -th component of x **component(x, n)**
 n -th component of vector/list x **x[n]**
 (m, n) -th component of matrix x **x[m, n]**
row m or column n of matrix x **x[m,], x[, n]**
numerator of x **numerator(x)**
lowest denominator of x **denominator(x)**

Conjugates and Lifts

conjugate of a number x **conj(x)**
conjugate vector of algebraic number x **conjvec(x)**
norm of x , product with conjugate **norm(x)**
square of L^2 norm of vector x **norml2(x)**
lift of x from Mods **lift, centerlift(x)**

Random Numbers

random integer between 0 and $N - 1$ **random({N})**
get random seed **getrand()**
set random seed to s **setrand(s)**

Lists, Sets & Sorting

sort x by k th component **vecsort($x, \{k\}, \{fl = 0\}$)**
Sets (= row vector of strings with strictly increasing entries)
intersection of sets x and y **setintersect(x, y)**
set of elements in x not belonging to y **setminus(x, y)**
union of sets x and y **setunion(x, y)**
look if y belongs to the set x **setsearch($x, y, \{flag\}$)**
Lists
create empty list L **L = List()**
append x to list L **listput($L, x, \{i\}$)**
remove i -th component from list L **listpop($L, \{i\}$)**
insert x in list L at position i **listinsert(L, x, i)**
sort the list L in place **listsort($L, \{flag\}$)**

Programming & User Functions

Control Statements (X : formal parameter in expression seq)
eval. seq for $a \leq X \leq b$ **for($X = a, b, seq$)**
eval. seq for X dividing n **fordiv(n, X, seq)**
eval. seq for primes $a \leq X \leq b$ **forprime($X = a, b, seq$)**
eval. seq for $a \leq X \leq b$ stepping s **forstep($X = a, b, s, seq$)**
multivariable for **forvec($X = v, seq$)**
if $a \neq 0$, evaluate seq_1 , else seq_2 **if($a, \{seq_1\}, \{seq_2\}$)**
evaluate seq until $a \neq 0$ **until(a, seq)**
while $a \neq 0$, evaluate seq **while(a, seq)**
exit n innermost enclosing loops **break({n})**
start new iteration of n th enclosing loop **next({n})**
return x from current subroutine **return({x})**
error recovery (try seq_1) **trap({err}, {seq₂}, {seq₁})**

Input/Output

print args with/without newline **print(), print1()**
formatted printing **printf()**
read a string from keyboard **input()**
output $args$ in T_EX format **printtex($args$)**
write $args$ to file **write, writel, writetex($file, args$)**
read file into GP **read({file})**

Interface with User and System

allocates a new stack of s bytes **allocatemem({s})**
execute system command a **system(a)**
as above, feed result to GP **extern(a)**
install function from library **install($f, code, \{gpf\}, \{lib\}$)**
alias old to new **alias(new, old)**
new name of function f in GP 2.0 **whatnow(f)**

User Defined Functions

name(formal vars) = my(local vars); seq
struct.member = seq
kill value of variable or function x **kill(x)**

Iterations, Sums & Products

numerical integration **intnum($X = a, b, expr, \{flag\}$)**
sum $expr$ over divisors of n **sumdiv($n, X, expr$)**
sum $X = a$ to $X = b$, initialized at x **sum($X = a, b, expr, \{x\}$)**
sum of series $expr$ **suminf($X = a, expr$)**
sum of alternating/positive series **sumalt, sumpos**
product $a \leq X \leq b$, initialized at x **prod($X = a, b, expr, \{x\}$)**
product over primes $a \leq X \leq b$ **prodeuler($X = a, b, expr$)**
infinite product $a \leq X \leq \infty$ **prodinf($X = a, expr$)**
real root of $expr$ between a and b **solve($X = a, b, expr$)**

Vectors & Matrices

| | |
|-----------------------------------|--|
| dimensions of matrix x | <code>matsize(x)</code> |
| concatenation of x and y | <code>concat($x, \{y\}$)</code> |
| extract components of x | <code>vecextract($x, y, \{z\}$)</code> |
| transpose of vector or matrix x | <code>mattranspose(x)</code> or <code>x-</code> |
| adjoint of the matrix x | <code>matadjoint(x)</code> |
| eigenvectors of matrix x | <code>mateigen(x)</code> |
| characteristic polynomial of x | <code>charpoly($x, \{v\}, \{flag\}$)</code> |
| minimal polynomial of x | <code>minpoly($x, \{v\}$)</code> |
| trace of matrix x | <code>trace(x)</code> |

Constructors & Special Matrices

| | |
|---|--|
| row vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vector($n, \{i\}, \{expr\}$)</code> |
| col. vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vectorv($n, \{i\}, \{expr\}$)</code> |
| matrix $1 \leq i \leq m, 1 \leq j \leq n$ | <code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code> |
| diagonal matrix with diagonal x | <code>matdiagonal(x)</code> |
| $n \times n$ identity matrix | <code>matid(n)</code> |
| Hessenberg form of square matrix x | <code>mathess(x)</code> |
| $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$ | <code>mathilbert(n)</code> |
| $n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$ | <code>matpascal($n - 1$)</code> |
| companion matrix to polynomial x | <code>matcompanion(x)</code> |

Gaussian elimination

| | |
|--|---|
| determinant of matrix x | <code>matdet($x, \{flag\}$)</code> |
| kernel of matrix x | <code>matker($x, \{flag\}$)</code> |
| intersection of column spaces of x and y | <code>matintersect(x, y)</code> |
| solve $M * X = B$ (M invertible) | <code>matsolve(M, B)</code> |
| as solve, modulo D (col. vector) | <code>matsolvemod(M, D, B)</code> |
| one sol of $M * X = B$ | <code>matinverseimage(M, B)</code> |
| basis for image of matrix x | <code>matimage(x)</code> |
| supplement columns of x to get basis | <code>mat supplement(x)</code> |
| rows, cols to extract invertible matrix | <code>matindexrank(x)</code> |
| rank of the matrix x | <code>matrank(x)</code> |

Lattices & Quadratic Forms

| | |
|--|--|
| upper triangular Hermite Normal Form | <code>mathnf(x)</code> |
| HNF of x where d is a multiple of $\det(x)$ | <code>mathnfmod(x, d)</code> |
| elementary divisors of x | <code>matsnf(x)</code> |
| LLL-algorithm applied to columns of x | <code>qflll($x, \{flag\}$)</code> |
| like qflll, x is Gram matrix of lattice | <code>qflllgram($x, \{flag\}$)</code> |
| LLL-reduced basis for kernel of x | <code>matkerint(x)</code> |
| \mathbf{Z} -lattice \longleftrightarrow \mathbf{Q} -vector space | <code>matrixqz(x, p)</code> |
| signature of quad form $t^y * x * y$ | <code>qfsign(x)</code> |
| decomp into squares of $t^y * x * y$ | <code>qfgaussred(x)</code> |
| find up to m sols of $t^y * x * y \leq b$ | <code>qfminim(x, b, m)</code> |
| $v, v[i] :=$ number of sols of $t^y * x * y = i$ | <code>qfrep($x, B, \{flag\}$)</code> |
| eigenvals/eigenvecs for real symmetric x | <code>qfjacobi(x)</code> |

Formal & p-adic Series

| | |
|---|--|
| truncate power series or p -adic number | <code>truncate(x)</code> |
| valuation of x at p | <code>valuation(x, p)</code> |
| Dirichlet and Power Series | |
| Taylor expansion around 0 of f w.r.t. x | <code>taylor(f, x)</code> |
| $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ | <code>serconvol(x, y)</code> |
| $f = \sum a_k t^k$ from $\sum (a_k / k!) t^k$ | <code>serlaplace(f)</code> |
| reverse power series F so $F(f(x)) = x$ | <code>serreverse(f)</code> |
| Dirichlet series multiplication / division | <code>dirmul, dirdiv(x, y)</code> |
| Dirichlet Euler product (b terms) | <code>direuler($p = a, b, expr$)</code> |

p-adic Functions

| | |
|-------------------------------------|--|
| Teichmuller character of x | <code>teichmuller(x)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

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Polynomials & Rational Functions

| | |
|---|--|
| degree of f | <code>poldegree(f)</code> |
| coefficient of degree n of f | <code>polcoeff(f, n)</code> |
| round coeffs of f to nearest integer | <code>round($f, \{\&e\}$)</code> |
| gcd of coefficients of f | <code>content(f)</code> |
| replace x by y in f | <code>subst(f, x, y)</code> |
| discriminant of polynomial f | <code>poldisc(f)</code> |
| resultant of f and g | <code>polresultant($f, g, \{v\}, \{flag\}$)</code> |
| as above, give $[u, v, d], xu + yv = d$ | <code>bezoutres(x, y)</code> |
| derivative of f w.r.t. x | <code>deriv(f, x)</code> |
| formal integral of f w.r.t. x | <code>intformal(f, x)</code> |
| reciprocal poly $x^{\deg f} f(1/x)$ | <code>polrecip(f)</code> |
| interpol. pol. eval. at a | <code>polinterpolate($X, \{Y\}, \{a\}, \{\&e\}$)</code> |
| initialize t for Thue equation solver | <code>thueinit(f)</code> |
| solve Thue equation $f(x, y) = a$ | <code>thue($t, a, \{sol\}$)</code> |

Roots and Factorization

| | |
|--|--|
| number of real roots of $f, a < x \leq b$ | <code>polsturm($f, \{a\}, \{b\}$)</code> |
| complex roots of f | <code>polroots(f)</code> |
| symmetric powers of roots of f up to n | <code>polsym(f, n)</code> |
| roots of f mod p | <code>polrootsmod($f, p, \{flag\}$)</code> |
| factor f | <code>factor($f, \{lim\}$)</code> |
| factorization of f mod p | <code>factormod($f, p, \{flag\}$)</code> |
| factorization of f over \mathbb{F}_{p^a} | <code>factorff(f, p, a)</code> |
| p -adic fact. of f to prec. r | <code>factorpadic($f, p, r, \{flag\}$)</code> |
| p -adic roots of f to prec. r | <code>polrootspadic(f, p, r)</code> |
| p -adic root of f cong. to a mod p | <code>padicappr(f, a)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

Special Polynomials

| | |
|--|--|
| n th cyclotomic polynomial in var. v | <code>polcyclo($n, \{v\}$)</code> |
| d -th degree subfield of $\mathbf{Q}(\zeta_n)$ | <code>polsubcyclo($n, d, \{v\}$)</code> |
| n -th Legendre polynomial | <code>pollegendre($n, \{v = x\}$)</code> |
| n -th Tchebicheff polynomial | <code>polchebyshev($n, \{flag\}, \{v = x\}$)</code> |
| Zagier's polynomial of index n, m | <code>polzagier(n, m)</code> |

Transcendental Functions

| | |
|--|--|
| real, imaginary part of x | <code>real(x), imag(x)</code> |
| absolute value, argument of x | <code>abs(x), arg(x)</code> |
| square/ n th root of x | <code>sqrtn($x, n, \{\&z\}$)</code> |
| trig functions | <code>sin, cos, tan, cotan</code> |
| inverse trig functions | <code>asin, acos, atan</code> |
| hyperbolic functions | <code>sinh, cosh, tanh</code> |
| inverse hyperbolic functions | <code>asinh, acosh, atanh</code> |
| exponential of x | <code>exp(x)</code> |
| natural log of x | <code>ln(x)</code> or <code>log(x)</code> |
| gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ | <code>gamma(x)</code> |
| logarithm of gamma function | <code>lngamma(x)</code> |
| $\psi(x) = \Gamma'(x) / \Gamma(x)$ | <code>psi(x)</code> |
| incomplete gamma function ($y = \Gamma(s)$) | <code>incgam($s, x, \{y\}$)</code> |
| exponential integral $\int_x^\infty e^{-t} / t dt$ | <code>eint1(x)</code> |
| error function $2 / \sqrt{\pi} \int_x^\infty e^{-t^2} dt$ | <code>erfc(x)</code> |
| dilogarithm of x | <code>dilog(x)</code> |
| m th polylogarithm of x | <code>polylog($m, x, \{flag\}$)</code> |
| U -confluent hypergeometric function | <code>hyperu(a, b, u)</code> |
| J -Bessel function, $J_{n+1/2}(x)$ | <code>besselj(n, x), besseljh(n, x)</code> |
| K -Bessel function of index nu | <code>besselk(nu, x)</code> |

Elementary Arithmetic Functions

| | |
|--|---|
| vector of binary digits of $ x $ | <code>binary(x)</code> |
| give bit number n of integer x | <code>bittest(x, n)</code> |
| ceiling of x | <code>ceil(x)</code> |
| floor of x | <code>floor(x)</code> |
| fractional part of x | <code>frac(x)</code> |
| round x to nearest integer | <code>round($x, \{\&e\}$)</code> |
| truncate x | <code>truncate($x, \{\&e\}$)</code> |
| gcd/LCM of x and y | <code>gcd(x, y), lcm(x, y)</code> |
| gcd of entries of a vector/matrix | <code>content(x)</code> |
| Primes and Factorization | |
| add primes in v to the prime table | <code>addprimes(v)</code> |
| the n th prime | <code>prime(n)</code> |
| vector of first n primes | <code>primes(n)</code> |
| smallest prime $\geq x$ | <code>nextprime(x)</code> |
| largest prime $\leq x$ | <code>precprime(x)</code> |
| factorization of x | <code>factor($x, \{lim\}$)</code> |
| reconstruct x from its factorization | <code>factorback($f, \{e\}$)</code> |

Divisors

| | |
|---|---|
| number of distinct prime divisors | <code>omega(x)</code> |
| number of prime divisors with mult | <code>bigomega(x)</code> |
| number of divisors of x | <code>numdiv(x)</code> |
| row vector of divisors of x | <code>divisors(x)</code> |
| sum of (k -th powers of) divisors of x | <code>sigma($x, \{k\}$)</code> |

Special Functions and Numbers

| | |
|--|--|
| binomial coefficient $\binom{x}{y}$ | <code>binomial(x, y)</code> |
| Bernoulli number B_n as real | <code>bernreal(n)</code> |
| Bernoulli vector B_0, B_2, \dots, B_{2n} | <code>bernvec(n)</code> |
| n th Fibonacci number | <code>fibonacci(n)</code> |
| number of partitions of n | <code>numbpart(n)</code> |
| Euler ϕ -function | <code>eulerphi(x)</code> |
| Möbius μ -function | <code>moebius(x)</code> |
| Hilbert symbol of x and y (at p) | <code>hilbert($x, y, \{p\}$)</code> |
| Kronecker-Legendre symbol $(\frac{x}{y})$ | <code>kronecker(x, y)</code> |

Miscellaneous

| | |
|--|--|
| integer or real factorial of x | <code>x!</code> or <code>fact(x)</code> |
| integer square root of x | <code>sqrtn(x)</code> |
| solve $z \equiv x$ and $z \equiv y$ | <code>chinese(x, y)</code> |
| minimal u, v so $xu + yv = \gcd(x, y)$ | <code>bezout(x, y)</code> |
| multiplicative order of x (intmod) (i=0) | <code>znorder($x, \{o\}$)</code> |
| primitive root mod prime power q | <code>znprimroot(q)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$ | <code>znstar(n)</code> |
| continued fraction of x | <code>contfrac($x, \{b\}, \{lmax\}$)</code> |
| last convergent of continued fraction x | <code>contfracpnqn(x)</code> |
| best rational approximation to x | <code>bestappr(x, k)</code> |

True-False Tests

| | |
|--|--|
| is x the disc. of a quadratic field? | <code>isfundamental(x)</code> |
| is x a prime? | <code>isprime(x)</code> |
| is x a strong pseudo-prime? | <code>ispseudoprime(x)</code> |
| is x square-free? | <code>issquarefree(x)</code> |
| is x a square? | <code>issquare($x, \{\&n\}$)</code> |
| is pol irreducible? | <code>polisirreducible(pol)</code> |

Based on an earlier version by Joseph H. Silverman
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PARI-GP Reference Card (2)

(PARI-GP version 2.5.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E=[a_1,a_2,a_3,a_4,a_6]$. Points are $[x,y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit($E,\{flag\}$)`

$a_1,a_2,a_3,a_4,a_6,b_2,b_4,b_6,b_8,c_4,c_6,disc,j$. This data can be recovered by typing `ell.a1,...,ell.j`. If $flag$ omitted, also

• E defined over \mathbf{R}

| | |
|-----------------------------------|------------------------|
| x -coords. of points of order 2 | <code>ell.roots</code> |
| real and complex periods | <code>ell.omega</code> |
| associated quasi-periods | <code>ell.eta</code> |
| volume of complex lattice | <code>ell.area</code> |

• E defined over \mathbf{Q}_p , $|j|_p > 1$

| | |
|-------------------------------------|------------------------|
| x -coord. of unit 2 torsion point | <code>ell.roots</code> |
| Tate's $[u^2,u,q]$ | <code>ell.tate</code> |
| Mestre's w | <code>ell.w</code> |

change curve E using $v=[u,r,s,t]$

`ellchangecurve(ell,v)`

change point z using $v=[u,r,s,t]$

`ellchangept(z,v)`

add points z_1+z_2

`elladd(ell,z_1,z_2)`

subtract points z_1-z_2

`ellsub(ell,z_1,z_2)`

compute $n \cdot z$

`ellpow(ell,z,n)`

check if z is on E

`ellisoncurve(ell,z)`

order of torsion point z

`ellorder(ell,z)`

y -coordinates of point(s) for x

`ellordinate(ell,x)`

point $[\wp(z),\wp'(z)]$ corresp. to z

`ellztopoint(ell,z)`

complex z such that $p=[\wp(z),\wp'(z)]$

`ellpointtoz(ell,p)`

Curves over finite fields, Pairings

random point on E

`random(ell)`

structure $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$ of $E(\mathbf{F}_p)$

`ellgroup(ell,p)`

Weil pairing of m -torsion pts x,y `ellweilpairing(ell,x,y,m)`

Tate pairing of x,y ; x m -torsion `elltatepairing(ell,x,y,m)`

Curves over \mathbf{Q} and the L -function

canonical bilinear form taken at z_1,z_2 `ellbil(ell,z_1,z_2)`

canonical height of z `ellheight($ell,z,\{flag\}$)`

height regulator matrix for pts in x `ellheightmatrix(ell,x)`

cond, min mod, Tamagawa num $[N,v,c]$ `ellglobalred(ell)`

Kodaira type of p -fiber of E `elllocalred(ell,p)`

minimal model of E/\mathbf{Q} `ellminimalmodel($ell,\{&v\}$)`

p th coeff a_p of L -function, p prime `ellap(ell,p)`

k th coeff a_k of L -function `ellak(ell,k)`

vector of first n a_k 's in L -function `ellan(ell,n)`

$L(E,s)$, set $A \approx 1$ `elllseries($ell,s,\{A\}$)`

order of vanishing at 1 `ellanalyticrank($ell,\{eps\}$)`

$L^{(r)}(E,1)$ `ellLi(ell,r)`

root number for $L(E,\cdot)$ at p `ellrootno($ell,\{p\}$)`

torsion subgroup with generators `elltors(ell)`

modular parametrization of E `elltaniyama(ell)`

Elldata package, Cremona's database:

db code \leftrightarrow $[conductor,class,index]$ `ellconvertname(s)`

generators of Mordell-Weil group `ellgenerators(E)`

look up E in database `ellidentify(E)`

all curves matching criterion `ellsearch(N)`

loop over curves with cond. from a to b `forell(E,a,b,seq)`

Elliptic & Modular Functions

arithmetic-geometric mean

`agm(x,y)`

elliptic j -function $1/q+744+\cdots$

`ellj(x)`

Weierstrass σ function

`ellsigma($ell,z,\{flag\}$)`

Weierstrass \wp function

`ellwp($ell,\{z\},\{flag\}$)`

Weierstrass ζ function

`ellzeta(ell,z)`

modified Dedekind η func. $\prod(1-q^n)$

`eta($x,\{flag\}$)`

Jacobi sine theta function

`theta(q,z)`

k -th derivative at $z=0$ of $\theta(q,z)$

`thetanulk(q,k)`

Weber's f functions

`weber($x,\{flag\}$)`

Riemann's zeta $\zeta(s)=\sum n^{-s}$

`zeta(s)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X=a,b,expr$)`

High-resolution plot (immediate plot)

plot $expr$ between a and b `ploto($X=a,b,expr,\{flag\},\{n\}$)`

plot points given by lists lx,ly `plotdraw($lx,ly,\{flag\}$)`

terminal dimensions

`plotsizes()`

Rectwindow functions

init window w , with size x,y

`plotinit(w,x,y)`

erase window w

`plotkill(w)`

copy w to w_2 with offset (dx,dy)

`plotcopy(w,w_2,dx,dy)`

scale coordinates in w

`plotscale(w,x_1,x_2,y_1,y_2)`

ploto in w `plotrecth($w,X=a,b,expr,\{flag\},\{n\}$)`

plotdraw in w `plotrectdraw($w,data,\{flag\}$)`

draw window w_1 at $(x_1,y_1),\dots$ `plotdraw([[w_1,x_1,y_1],...])`

Low-level Rectwindow Functions

set current drawing color in w to c

`plotcolor(w,c)`

current position of cursor in w

`plotcursor(w)`

write s at cursor's position

`plotstring(w,s)`

move cursor to (x,y)

`plotmove(w,x,y)`

move cursor to $(x+dx,y+dy)$

`plotrmove(w,dx,dy)`

draw a box to (x_2,y_2)

`plotbox(w,x_2,y_2)`

draw a box to $(x+dx,y+dy)$

`plotrbox(w,dx,dy)`

draw polygon

`plotlines($w,lx,ly,\{flag\}$)`

draw points

`plotpoints(w,lx,ly)`

draw line to $(x+dx,y+dy)$

`plotrline(w,dx,dy)`

draw point $(x+dx,y+dy)$

`plotrpoint(w,dx,dy)`

Postscript Functions

as ploto `psploto($X=a,b,expr,\{flag\},\{n\}$)`

as plotdraw `psplotdraw($lx,ly,\{flag\}$)`

as plotdraw `psdraw([[w_1,x_1,y_1],...])`

Binary Quadratic Forms

create $ax^2+bxy+cy^2$ (distance d) `qfb($a,b,c,\{d\}$)`

reduce x ($s=\sqrt{D}$, $l=\lfloor s \rfloor$) `qfbred($x,\{flag\},\{D\},\{l\},\{s\}$)`

composition of forms $x*y$ or `qfbnucomp(x,y,l)`

n -th power of form x^n or `qfbnpow(x,n)`

composition without reduction `qfbcomprow(x,y)`

n -th power without reduction `qfbpowrow(x,n)`

prime form of disc. x above prime p `qfbprimeform(x,p)`

class number of disc. x `qfbclassno(x)`

Hurwitz class number of disc. x `qfbhclassno(x)`

Quadratic Fields

quadratic number $\omega=\sqrt{x}$ or $(1+\sqrt{x})/2$ `quadgen(x)`

minimal polynomial of ω `quadpoly(x)`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc(x)`

regulator of real quadratic field `quadregulator(x)`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit(x)`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit($D,\{flag\},\{t\}$)`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert($D,\{flag\}$)`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadray($D,f,\{flag\}$)`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit($f,\{flag\}$)`

nf members:

polynomial defining nf , $f(\theta)=0$ `nf.pol`

number of real/complex places `nf.r1/r2/sign`

discriminant of nf `nf.disc`

T_2 matrix `nf.t2`

vector of roots of f `nf.roots`

integral basis of \mathbf{Z}_K as powers of θ `nf.zk`

different `nf.diff`

codifferent `nf.codiff`

index `nf.index`

recompute nf using current precision `nfnewprec(nf)`

init relative rnf given by $g=0$ over K `rnfinit(nf,g)`

init bnf structure `bnfinit($f,\{flag\}$)`

bnf members: same as nf , plus

underlying nf `bnf.nf`

classgroup `bnf.clgp`

regulator `bnf.reg`

fundamental units `bnf.fu`

torsion units `bnf.tu`

compute a bnf from small bnf `bnfinit($sbnf$)`

add S -class group and units, yield bnf s `bnfsunit(nf,S)`

init class field structure bnr `bnrinit($bnf,m,\{flag\}$)`

bnr members: same as bnf , plus

underlying bnf `bnr.bnf`

big ideal structure `bnr.bid`

modulus `bnr.mod`

structure of $(\mathbf{Z}_K/m)^*$ `bnr.zkst`

Basic Number Field Arithmetic (nf)

Elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis `nf.zk`). Basic operations (prefix `nfelt`): `(nfelt)add`, `mul`, `pow`, `div`, `diveuc`, `mod`, `divrem`, `val`, `trace`, `norm`
express x on integer basis `nfalgtobasis(nf, x)`
express element x as a polmod `nfbasistoalg(nf, x)`
reverse polmod $a = A(X) \bmod T(X)$ `modreverse(a)`
integral basis of field def. by $f = 0$ `nfbasis(f)`
field discriminant of field $f = 0$ `nfdisc(f)`
Galois group of field $f = 0$, $\deg f \leq 11$ `polgalois(f)`
smallest poly defining $f = 0$ `polredabs(f, {flag})`
small polys defining subfields of $f = 0$ `polred(f, {flag}, {p})`
poly of degree $\leq k$ with root $x \in \mathbf{C}$ `algdep(x, k)`
small linear rel. on coords of vector x `lindep(x)`
are fields $f = 0$ and $g = 0$ isomorphic? `nfisom(f, g)`
is field $f = 0$ a subfield of $g = 0$? `nfisincl(f, g)`
compositum of $f = 0$, $g = 0$ `polcompositum(f, g, {flag})`
subfields (of degree d) of nf `nfsubfields(nf, {d})`
roots of unity in nf `nfrootsof1(nf)`
roots of g belonging to nf `nfroots({nf}, g)`
factor g in nf `nfactor(nf, g)`
factor g mod prime pr in nf `nfactormod(nf, g, pr)`
conjugates of a root θ of nf `nfgaloisconj(nf, {flag})`
apply Galois automorphism s to x `nfgaloisapply(nf, s, x)`
quadratic Hilbert symbol (at p) `nfhilbert(nf, a, b, {p})`
Dedekind Zeta Function ζ_K
 ζ_K as Dirichlet series, $N(I) < b$ `dirzetak(nf, b)`
init nfz for field $f = 0$ `zetakinit(f)`
compute $\zeta_K(s)$ `zetak(nfz, s, {flag})`
Artin root number of K `bnrrootnumber(bnr, chi, {flag})`

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually bnr , $subgp$ or bnf , $module$, $\{subgp\}$
remove GRH assumption from bnf `bnfcertify(bnf)`
expo. of ideal x on class gp `bnfisprincipal(bnf, x, {flag})`
expo. of ideal x on ray class gp `bnrisprincipal(bnr, x, {flag})`
expo. of x on fund. units `bnfisunit(bnf, x)`
as above for S -units `bnfissunit(bnfs, x)`
signs of real embeddings of bnf .fu `bnfsignunit(bnf)`

Class Field Theory

ray class number for mod. m `bnrclassno(bnf, m)`
discriminant of class field ext `bnrdisc(a1, {a2}, {a3})`
ray class numbers, l list of mods `bnrclassnolist(bnf, l)`
discriminants of class fields `bnrdisclist(bnf, l, {arch}, {flag})`
decode output from `bnrdisc` `bnfdecodemodule(nf, fa)`
is modulus the conductor? `bnrisconductor(a1, {a2}, {a3})`
conductor of character chi `bnrconductorofchar(bnr, chi)`
conductor of extension `bnrconductor(a1, {a2}, {a3}, {flag})`
conductor of extension def. by g `rnfconductor(bnf, g)`
Artin group of ext. def'd by g `rnfnormgroup(bnr, g)`
subgroups of bnr , index $\leq b$ `subgrouplist(bnr, b, {flag})`
rel. eq. for class field def'd by sub `rnfkummer(bnr, sub, {d})`
same, using Stark units (real field) `bnrstark(bnr, sub, {flag})`

PARI-GP Reference Card (2)

(PARI-GP version 2.5.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
is id an ideal in nf ? `nfisideal(nf, id)`
is x principal in bnf ? `bnfisprincipal(bnf, x)`
principal ideal generated by x `idealprincipal(nf, x)`
principal idele generated by x `ideleprincipal(nf, x)`
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ `idealtwoelt(nf, x, {a})`
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form `idealhnf(nf, a, {b})`
norm of ideal x `idealnrm(nf, x)`
minimum of ideal x (direction v) `idealmin(nf, x, v)`
LLL-reduce the ideal x (direction v) `idealred(nf, x, {v})`

Ideal Operations

add ideals x and y `idealadd(nf, x, y)`
multiply ideals x and y `idealmul(nf, x, y, {flag})`
intersection of ideals x and y `idealintersect(nf, x, y, {flag})`
 n -th power of ideal x `idealpow(nf, x, n, {flag})`
inverse of ideal x `idealinv(nf, x)`
divide ideal x by y `idealdiv(nf, x, y, {flag})`
Find $(a, b) \in x \times y$, $a + b = 1$ `idealaddtoone(nf, x, {y})`

Primes and Multiplicative Structure

factor ideal x in nf `idealfactor(nf, x)`
expand ideal factorization in nf `idealfactorback(nf, f, e)`
decomposition of prime p in nf `idealprimedec(nf, p)`
valuation of x at prime ideal pr `idealval(nf, x, pr)`
weak approximation theorem in nf `idealchinese(nf, x, y)`
give bid = structure of $(\mathbf{Z}_K/id)^*$ `idealstar(nf, id, {flag})`
discrete log of x in $(\mathbf{Z}_K/bid)^*$ `ideallog(nf, x, bid)`
idealstar of all ideals of norm $\leq b$ `ideallist(nf, b, {flag})`
add Archimedean places `ideallistarch(nf, b, {ar}, {flag})`
init `prmod` structure `nfmodprinit(nf, pr)`
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ `nfkermodpr(nf, M, prmod)`
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ `nfsolvemodpr(nf, M, B, prmod)`

Galois theory over \mathbf{q}

initializes a Galois group structure `galoisinit(pol, {den})`
action of p in `nfgaloisconj` form `galoispermopol(G, {p})`
identifies as abstract group `galoisidentify(G)`
exports a group for GAP or MAGMA `galoisexport(G, {flag})`
subgroups of the Galois group G `galoissubgroups(G)`
subfields from subgroups of G `galoissubfields(G, {flag}, {v})`
fixed field `galoisfixedfield(G, perm, {flag}, {v})`
is G abelian? `galoisisabelian(G, {flag})`
abelian number fields `galoissubcyclo(N, H, {flag}, {v})`

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
absolute equation of L `rnfequation(nf, g, {flag})`
relative `nfalgtobasis` `rnfalgtobasis(rnf, x)`
relative `nfbasistoalg` `rnfbasistoalg(rnf, x)`
relative `idealhnf` `rnfidealhnf(rnf, x)`
relative `idealmul` `rnfidealmul(rnf, x, y)`
relative `idealtwoelt` `rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute \rightarrow relative repres. for x `rnfeltabstorel(rnf, x)`
relative \rightarrow absolute repres. for x `rnfeltreltoabs(rnf, x)`
lift x to the relative field `rnfeltup(rnf, x)`
push x down to the base field `rnfeltdown(rnf, x)`
idem for x ideal: `(rnfideal)reltoabs, astorel, up, down`

Projective \mathbf{Z}_K -modules, maximal order

relative `polred` `rnfpolred(nf, g)`
relative `polredabs` `rnfpolredabs(nf, g)`
characteristic poly. of $a \bmod g$ `rnfcharpoly(nf, g, a, {v})`
relative Dedekind criterion, prime pr `rnfdedekind(nf, g, pr)`
discriminant of relative extension `rnfdisc(nf, g)`
pseudo-basis of \mathbf{Z}_L `rnfpseudobasis(nf, g)`
relative HNF basis of $order$ `rnfhnfbasis(bnf, order)`
reduced basis for $order$ `rnflllgram(nf, g, order)`
determinant of pseudo-matrix A `rnfdet(nf, A)`
Steinitz class of $order$ `rnfsteynitz(nf, order)`
is $order$ a free \mathbf{Z}_K -module? `rnfisfree(bnf, order)`
true basis of $order$, if it is free `rnfbasis(bnf, order)`

Norms

absolute norm of ideal x `rnfidealnrmabs(rnf, x)`
relative norm of ideal x `rnfidealnrmrel(rnf, x)`
solutions of $N_K/\mathbf{Q}(y) = x \in \mathbf{Z}$ `bnfisintnorm(bnf, x)`
is $x \in \mathbf{Q}$ a norm from K ? `bnfisnorm(bnf, x, {flag})`
initialize T for norm eq. solver `rnfisnorminit(K, pol, {flag})`
is $a \in K$ a norm from L ? `rnfisnorm(T, a, {flag})`

Based on an earlier version by Joseph H. Silverman
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