

# Pari-GP reference card

(PARI-GP version 2.13.4)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function	? <i>function</i>
extended description	?? <i>keyword</i>
list of relevant help topics	??? <i>pattern</i>
name of GP-1.39 function <i>f</i> in GP-2.*	whatnow( <i>f</i> )

## Input/Output

previous result, the result before	%, %`, %``, etc.
<i>n</i> -th result since startup	% <i>n</i>
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ <i>seq</i> <sub>1</sub> ; <i>seq</i> <sub>2</sub> ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

## Metacommands & Defaults

set default <i>d</i> to <i>val</i>	default({ <i>d</i> },{ <i>val</i> })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to <i>n</i>	\g <i>n</i>
set memory debug level to <i>n</i>	\gm <i>n</i>
set <i>n</i> significant digits / bits	\p <i>n</i> , \pb <i>n</i>
set <i>n</i> terms in series	\ps <i>n</i>
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

## Debugger / break loop

get out of break loop	break or <C-D>
go up/down <i>n</i> frames	dbg_up({ <i>n</i> }), dbg_down
set break point	breakpoint()
examine object <i>o</i>	dbg_x( <i>o</i> )
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

## PARI Types & Input Formats

t_INT. Integers; hex, binary	±31; ±0x1F, ±0b101
t_REAL. Reals	±3.14, 6.022 E23
t_INTMOD. Integers modulo <i>m</i>	Mod( <i>n</i> , <i>m</i> )
t_FRAC. Rational Numbers	<i>n</i> / <i>m</i>
t_FFELT. Elt in finite field <b>F</b> <sub><i>q</i></sub>	ffgen( <i>q</i> , 't)
t_COMPLEX. Complex Numbers	<i>x</i> + <i>y</i> * I
t_PADIC. <i>p</i> -adic Numbers	<i>x</i> + 0( <i>p</i> <sup><i>k</i></sup> )
t_QUAD. Quadratic Numbers	<i>x</i> + <i>y</i> * quadgen( <i>D</i> , {'w'})
t_POLMOD. Polynomials modulo <i>g</i>	Mod( <i>f</i> , <i>g</i> )
t_POL. Polynomials	<i>a</i> * <i>x</i> <sup><i>n</i></sup> + ... + <i>b</i>
t_SER. Power Series	<i>f</i> + 0( <i>x</i> <sup><i>k</i></sup> )
t_RFRAC. Rational Functions	<i>f</i> / <i>g</i>
t_QFI/t_QFR. Imag/Real binary quad. form	Qfb( <i>a</i> , <i>b</i> , <i>c</i> , { <i>d</i> })
t_VEC/t_COL. Row/Column Vectors	[ <i>x</i> , <i>y</i> , <i>z</i> ], [ <i>x</i> , <i>y</i> , <i>z</i> ]~
t_VEC integer range	[1..10]

t_VECSMALL. Vector of small ints	Vecsmall([ <i>x</i> , <i>y</i> , <i>z</i> ])
t_MAT. Matrices	[ <i>a</i> , <i>b</i> ; <i>c</i> , <i>d</i> ]
t_LIST. Lists	List([ <i>x</i> , <i>y</i> , <i>z</i> ])
t_STR. Strings	"abc"
t_INFINITY. ±∞	+oo, -oo

## Reserved Variable Names

$\pi \approx 3.14$ , $\gamma \approx 0.57$ , $C \approx 0.91$ , $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

## Information about an Object, Precision

PARI type of object <i>x</i>	type( <i>x</i> )
length of <i>x</i> / size of <i>x</i> in memory	# <i>x</i> , sizebyte( <i>x</i> )
real precision / bit precision of <i>x</i>	precision( <i>x</i> ), bitprecision( <i>x</i> )
<i>p</i> -adic, series prec. of <i>x</i>	padicprec( <i>x</i> , <i>p</i> ), serprec( <i>x</i> , <i>v</i> )
current dynamic precision	getlocalprec, getlocalbitprec

## Operators

basic operations	+, −, *, /, ^, sqr
$i \leftarrow i+1$ , $i \leftarrow i-1$ , $i \leftarrow i*j$ , ...	i++, i--, i*=j,...
Euclidean quotient, remainder	<i>x</i> \/ <i>y</i> , <i>x</i> / <i>y</i> , <i>x</i> % <i>y</i> , divrem( <i>x</i> , <i>y</i> )
shift <i>x</i> left or right <i>n</i> bits	<i>x</i> << <i>n</i> , <i>x</i> >> <i>n</i> or shift( <i>x</i> ,± <i>n</i> )
multiply by 2 <sup><i>n</i></sup>	shiftmul( <i>x</i> , <i>n</i> )
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of <i>x</i> and <i>y</i>	max( <i>x</i> , <i>y</i> ), min( <i>x</i> , <i>y</i> )
sign of <i>x</i> (gives −1, 0, 1)	sign( <i>x</i> )
binary exponent of <i>x</i>	exponent( <i>x</i> )
derivative of <i>f</i> , 2nd derivative, etc.	<i>f</i> ' , <i>f</i> '', ...
differential operator	diffop( <i>f</i> , <i>v</i> , <i>d</i> , { <i>n</i> = 1})
quote operator (formal variable)	'x
assignment	x = <i>value</i>
simultaneous assignment $x \leftarrow v[1]$ , $y \leftarrow v[2]$	[x,y] = v

## Select Components

*Caveat*: components start at index *n* = 1.

<i>n</i> -th component of <i>x</i>	component( <i>x</i> , <i>n</i> )
<i>n</i> -th component of vector/list <i>x</i>	<i>x</i> [ <i>n</i> ]
components <i>a</i> , <i>a</i> + 1, ..., <i>b</i> of vector <i>x</i>	<i>x</i> [ <i>a</i> .. <i>b</i> ]
( <i>m</i> , <i>n</i> )-th component of matrix <i>x</i>	<i>x</i> [ <i>m</i> , <i>n</i> ]
row <i>m</i> or column <i>n</i> of matrix <i>x</i>	<i>x</i> [ <i>m</i> ,], <i>x</i> [, <i>n</i> ]
numerator/denominator of <i>x</i>	numerator( <i>x</i> ), denominator( <i>x</i> )

## Random Numbers

random integer/prime in [0, <i>N</i> [	random( <i>N</i> ), randomprime( <i>N</i> )
get/set random seed	getrand, setrand( <i>s</i> )

## Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create ( <i>x</i> mod <i>y</i> )	Mod( <i>x</i> , <i>y</i> )
make <i>x</i> a polynomial of <i>v</i>	Pol( <i>x</i> , { <i>v</i> })
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make <i>x</i> a power series of <i>v</i>	Ser( <i>x</i> , { <i>v</i> })
convert <i>x</i> to simplest possible type	simplify( <i>x</i> )
object <i>x</i> with real precision <i>n</i>	precision( <i>x</i> , <i>n</i> )
object <i>x</i> with bit precision <i>n</i>	bitprecision( <i>x</i> , <i>n</i> )
set precision to <i>p</i> digits in dynamic scope	localprec( <i>p</i> )
set precision to <i>p</i> bits in dynamic scope	localbitprec( <i>p</i> )

## Character strings

convert to TeX representation	strtex( <i>x</i> )
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time <i>t</i> ms. to h, m, s, ms format	strtime( <i>t</i> )
<b>Conjugates and Lifts</b>	
conjugate of a number <i>x</i>	conj( <i>x</i> )
norm of <i>x</i> , product with conjugate	norm( <i>x</i> )
$L^p$ norm of <i>x</i> ( $L^\infty$ if no <i>p</i> )	normlp( <i>x</i> , { <i>p</i> })
square of $L^2$ norm of <i>x</i>	norml2( <i>x</i> )
lift of <i>x</i> from Mods and <i>p</i> -adics	lift, centerlift( <i>x</i> )
recursive lift	liftall
lift all t_INT and t_PADIC ( $\rightarrow$ t_INT)	liftint
lift all t_POLMOD ( $\rightarrow$ t_POL)	lifttpol

## Lists, Sets & Maps

<b>Sets</b> (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets <i>x</i> and <i>y</i>	setintersect( <i>x</i> , <i>y</i> )
set of elements in <i>x</i> not belonging to <i>y</i>	setminus( <i>x</i> , <i>y</i> )
union of sets <i>x</i> and <i>y</i>	setunion( <i>x</i> , <i>y</i> )
does <i>y</i> belong to the set <i>x</i>	setsearch( <i>x</i> , <i>y</i> , { <i>flag</i> })
set of all <i>f</i> ( <i>x</i> , <i>y</i> ), $x \in X$ , $y \in Y$	setbinop( <i>f</i> , <i>X</i> , <i>Y</i> )
is <i>x</i> a set ?	setisset( <i>x</i> )

<b>Lists.</b> create empty list: <i>L</i> = List()	
append <i>x</i> to list <i>L</i>	listput( <i>L</i> , <i>x</i> , { <i>i</i> })
remove <i>i</i> -th component from list <i>L</i>	listpop( <i>L</i> , { <i>i</i> })
insert <i>x</i> in list <i>L</i> at position <i>i</i>	listinsert( <i>L</i> , <i>x</i> , <i>i</i> )
sort the list <i>L</i> in place	listsort( <i>L</i> , { <i>flag</i> })

<b>Maps.</b> create empty dictionary: <i>M</i> = Map()	
attach value <i>v</i> to key <i>k</i>	mapput( <i>M</i> , <i>k</i> , <i>v</i> )
recover value attach to key <i>k</i> or error	mapget( <i>M</i> , <i>k</i> )
is key <i>k</i> in the dict? (set <i>v</i> to <i>M</i> ( <i>k</i> ))	mapisdefined( <i>M</i> , <i>k</i> , {& <i>v</i> })
remove <i>k</i> from map domain	mapdelete( <i>M</i> , <i>k</i> )

## GP Programming

### User functions and closures

*x*, *y* are formal parameters; *y* defaults to Pi if parameter omitted; *z*, *t* are local variables (lexical scope), *z* initialized to 1.

fun(x, y=Pi) = my(z=1, t); seq  
fun = (x, y=Pi) -> my(z=1, t); seq

attach help message <i>h</i> to <i>s</i>	addhelp( <i>s</i> , <i>h</i> )
undefine symbol <i>s</i> (also kills help)	kill( <i>s</i> )
<b>Control Statements</b> ( <i>X</i> : formal parameter in expression <i>seq</i> )	
if <i>a</i> ≠ 0, evaluate <i>seq</i> <sub>1</sub> , else <i>seq</i> <sub>2</sub>	if( <i>a</i> , { <i>seq</i> <sub>1</sub> }, { <i>seq</i> <sub>2</sub> })
eval. <i>seq</i> for $a \leq X \leq b$	for( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
...for <i>X</i> ∈ <i>v</i>	foreach( <i>v</i> , <i>X</i> , <i>seq</i> )
...for primes $a \leq X \leq b$	forprime( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
...for primes $\equiv a \pmod{q}$	forprimestep( <i>X</i> = <i>a</i> , <i>b</i> , <i>q</i> , <i>seq</i> )
...for composites $a \leq X \leq b$	forcomposite( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
...for $a \leq X \leq b$ stepping <i>s</i>	forstep( <i>X</i> = <i>a</i> , <i>b</i> , <i>s</i> , <i>seq</i> )
...for <i>X</i> dividing <i>n</i>	fordiv( <i>n</i> , <i>X</i> , <i>seq</i> )
... <i>X</i> = [ <i>n</i> , factor( <i>n</i> )], $a \leq n \leq b$	forfactored( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
...as above, <i>n</i> squarefree	forsquarefree( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
... <i>X</i> = [ <i>d</i> , factor( <i>d</i> )], <i>d</i>   <i>n</i>	fordivfactored( <i>n</i> , <i>X</i> , <i>seq</i> )
multivariable for, lex ordering	forvec( <i>X</i> = <i>v</i> , <i>seq</i> )

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v, q(v) \leq B; q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ , TeX format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqfvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsrt( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{\&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()
write, writel, writetex( $file, args$ )
writeln( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

# Pari-GP reference card

(PARI-GP version 2.13.4)

## Timers

CPU time in  $ms$  and reset timer  
CPU time in  $ms$  since gp startup  
time in  $ms$  since UNIX Epoch  
timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes  
alias  $old$  to  $new$   
install function from library  
execute system command  $a$   
... and feed result to GP  
... returning GP string  
get \$VAR from environment  
expand env. variable in string

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate  $f$  on  $x[1], \dots, x[n]$   
evaluate closures  $f[1], \dots, f[n]$

as **select**

as **sum**

as **vector**

eval  $f$  for  $i = a, \dots, b$

... for each element  $x$  in  $v$

... for  $p$  prime in  $[a, b]$

... for  $p = a \bmod q$

... multivariate

export  $x$  to parallel world

... all dynamic variables

frees exported value  $x$

... all exported values

## Linear Algebra

dimensions of matrix  $x$

multiply two matrices

... assuming result is diagonal

concatenation of  $x$  and  $y$

extract components of  $x$

transpose of vector or matrix  $x$

adjoint of the matrix  $x$

eigenvectors/values of matrix  $x$

characteristic/minimal polynomial of  $x$

trace/determinant of matrix  $x$

permanent of matrix  $x$

Frobenius form of  $x$

QR decomposition

apply **matqr**'s transform to  $v$

## Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$

$\{x: x \in v \text{ s.t. } f(x)\}$

$\{g(x): x \in v\}$

row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$

col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$

vector of small ints

**gettime**()

**getabstime**()

**getwalltime**()

**alarm**( $s, expr$ )

**allocatemem**( $\{s\}$ )

**alias**( $new, old$ )

**install**( $f, code, \{gpf\}, \{lib\}$ )

**system**( $a$ )

**extern**( $a$ )

**externstr**( $a$ )

**getenv**("VAR")

**strex**and( $x$ )

**parapply**( $f, x$ )

**pareval**( $f$ )

**parselect**( $f, A, \{flag\}$ )

**parsum**( $i = a, b, expr$ )

**parvector**( $n, i, \{expr\}$ )

**parfor**( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )

**parforeach**( $v, x, f, \{r\}, \{f_2\}$ )

**parforprime**( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )

**parforprimestep**( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )

**parforvec**( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )

**export**( $x$ )

**exportall**()

**unexport**( $x$ )

**unexportall**()

**matsize**( $x$ )

$x * y$

**matmultodiagonal**( $x, y$ )

**concat**( $x, \{y\}$ )

**vecextract**( $x, y, \{z\}$ )

$x \sim$ , **mattranspose**( $x$ )

**matadjoint**( $x$ )

**mateigen**( $x$ )

**charpoly**( $x$ ), **minpoly**( $x$ )

**trace**( $x$ ), **matdet**( $x$ )

**matpermanent**( $x$ )

**matfrobenius**( $x$ )

**matqr**( $x$ )

**mathouseholder**( $Q, v$ )

$[g(x) \mid x \leftarrow v, f(x)]$

$[x \mid x \leftarrow v, f(x)]$

$[g(x) \mid x \leftarrow v]$

**vector**( $n, \{i\}, \{expr\}$ )

**vectorv**( $n, \{i\}, \{expr\}$ )

**vectorsmall**( $n, \{i\}, \{expr\}$ )

$[c, c \cdot x, \dots, c \cdot x^n]$

$[1, 2^x, \dots, n^x]$

matrix  $1 \leq i \leq m, 1 \leq j \leq n$

define matrix by blocks

diagonal matrix with diagonal  $x$

is  $x$  diagonal?

$x \cdot \text{matdiagonal}(d)$

$n \times n$  identity matrix

Hessenberg form of square matrix  $x$

$n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$

$n \times n$  Pascal triangle

companion matrix to polynomial  $x$

Sylvester matrix of  $x$  and  $y$

## Gaussian elimination

kernel of matrix  $x$

intersection of column spaces of  $x$  and  $y$

solve  $MX = B$  ( $M$  invertible)

one sol of  $M * X = B$

basis for image of matrix  $x$

columns of  $x$  *not* in **matimage**

supplement columns of  $x$  to get basis

rows, cols to extract invertible matrix

rank of the matrix  $x$

solve  $MX = B \bmod D$

image mod  $D$

kernel mod  $D$

inverse mod  $D$

determinant mod  $D$

## Lattices & Quadratic Forms

### Quadratic forms

evaluate  ${}^t x Q y$

evaluate  ${}^t x Q x$

signature of quad form  ${}^t y * x * y$

decomp into squares of  ${}^t y * x * y$

eigenvalues/vectors for real symmetric  $x$

### HNF and SNF

upper triangular Hermite Normal Form

HNF of  $x$  where  $d$  is a multiple of  $\det(x)$

multiple of  $\det(x)$

HNF of  $(x \mid \text{diagonal}(D))$

elementary divisors of  $x$

elementary divisors of  $\mathbf{Z}[a]/(f'(a))$

integer kernel of  $x$

**Z**-module  $\leftrightarrow$  **Q**-vector space

### Lattices

LLL-algorithm applied to columns of  $x$

... for Gram matrix of lattice

find up to  $m$  sols of **qfnorm**( $x, y) \leq b$

$v, v[i] :=$  number of  $y$  s.t. **qfnorm**( $x, y) = i$

perfection rank of  $x$

find isomorphism between  $q$  and  $Q$

precompute for isomorphism test with  $q$

automorphism group of  $q$

**powers**( $x, n, \{c = 1\}$ )

**dirpowers**( $n, x$ )

**matrix**( $m, n, \{i\}, \{j\}, \{expr\}$ )

**matconcat**( $B$ )

**matdiagonal**( $x$ )

**matisdiagonal**( $x$ )

**matmuldiagonal**( $x, d$ )

**matid**( $n$ )

**mathess**( $x$ )

**mathilbert**( $n$ )

**matpascal**( $n - 1$ )

**matcompanion**( $x$ )

**polsylvestermatrix**( $x, y$ )

**matker**( $x, \{flag\}$ )

**matintersect**( $x, y$ )

**matsolve**( $M, B$ )

**matinverseimage**( $M, B$ )

**matimage**( $x$ )

**matimagecompl**( $x$ )

**matsupplement**( $x$ )

**matindexrank**( $x$ )

**matrank**( $x$ )

**matsolvemod**( $M, D, B$ )

**matimagemod**( $M, D$ )

**matkernelmod**( $M, D$ )

**matinvmod**( $M, D$ )

**matdetmod**( $M, D$ )

**qfeval**( $\{Q = id\}, x, y$ )

**qfeval**( $\{Q = id\}, x$ )

**qfsign**( $x$ )

**qfgaussred**( $x$ )

**qfjacobi**( $x$ )

**mathnf**( $x$ )

**mathnfmod**( $x, d$ )

**matdetint**( $x$ )

**mathnfmodid**( $x, D$ )

**matsnf**( $x$ )

**poldiscred**uced( $f$ )

**matkerint**( $x$ )

**matrixqz**( $x, p$ )

**qflll**( $x, \{flag\}$ )

**qflllgram**( $x, \{flag\}$ )

**qfminim**( $x, b, m$ )

**qfrep**( $x, B, \{flag\}$ )

**qfperfection**( $x$ )

**qfisom**( $q, Q$ )

**qfisominit**( $q$ )

**qfauto**( $q$ )

Based on an earlier version by Joseph H. Silverman

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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert `qfauto` for GAP/Magma `qfautoexport( $G, \{flag\}$ )`  
orbits of  $V$  under  $G \subset \mathrm{GL}(V)$  `qforbits( $G, V$ )`

Polynomials & Rational Functions

all defined polynomial variables `variables()`  
get var. of highest priority (higher than  $v$ ) `varhigher( $name, \{v\}$ )`  
... of lowest priority (lower than  $v$ ) `varlower( $name, \{v\}$ )`

Coefficients, variables and basic operators

degree of  $f$  `poldegree( $f$ )`  
coef. of degree  $n$  of  $f$ , leading coef. `polcoef( $f, n$ ), pollead`  
main variable / all variables in  $f$  `variable( $f$ ), variables( $f$ )`  
replace  $x$  by  $y$  in  $f$  `subst( $f, x, y$ )`  
evaluate  $f$  replacing vars by their value `eval( $f$ )`  
replace polynomial expr.  $T(x)$  by  $y$  in  $f$  `substpol( $f, T, y$ )`  
replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$  `substvec( $f, x, y$ )`

$f \in A[x]$ ; reciprocal polynomial  $x^{\deg f} f\left(\frac{1}{x}\right)$  `polrecip( $f$ )`  
gcd of coefficients of  $f$  `content( $f$ )`  
derivative of  $f$  w.r.t.  $x$  `deriv( $f, \{x\}$ )`  
...  $n$ -th derivative of  $f$  `derivn( $f, n, \{x\}$ )`  
formal integral of  $f$  w.r.t.  $x$  `intformal( $f, \{x\}$ )`  
formal sum of  $f$  w.r.t.  $x$  `sumformal( $f, \{x\}$ )`

Constructors & Special Polynomials

interpolation polynomial at  $(x[1], y[1]), \dots, (x[n], y[n])$ , evaluated at  $t$ , with error estimate  $e$  `polinterpolate( $x, \{y\}, \{t\}, \{&e\}$ )`  
 $T_n/U_n, H_n$  `polchebyshev( $n$ ), polhermite( $n$ )`  
 $P_n, L_n^{(\alpha)}$  `pollegendre( $n$ ), pollaguerre( $n, a$ )`  
 $n$ -th cyclotomic polynomial  $\Phi_n$  `polcyclo( $n$ )`  
return  $n$  if  $f = \Phi_n$ , else 0 `poliscyclo( $f$ )`  
is  $f$  a product of cyclotomic polynomials? `poliscycloprod( $f$ )`  
Zagier's polynomial of index  $(n, m)$  `polzagier( $n, m$ )`

Resultant, elimination

discriminant of polynomial  $f$  `poldisc( $f$ )`  
find factors of `poldisc( $f$ )` `polresultant( $f, g, \{v\}$ )`  
resultant  $R = \mathrm{Res}_v(f, g)$  `polresultanttext( $x, y, \{v\}$ )`  
 $[u, v, R], xu + yv = \mathrm{Res}_v(f, g)$   
solve Thue equation  $f(x, y) = a$  `thue( $t, a, \{sol\}$ )`  
initialize  $t$  for Thue equation solver `thueinit( $f$ )`

Roots and Factorization (Complex/Real)

complex roots of  $f$  `polroots( $f$ )`  
bound complex roots of  $f$  `polrootsbound( $f$ )`  
number of real roots of  $f$  (in  $[a, b]$ ) `polsturm( $f, \{[a, b]\}$ )`  
real roots of  $f$  (in  $[a, b]$ ) `polrootsreal( $f, \{[a, b]\}$ )`  
complex embeddings of `t_POLMOD`  $z$  `conjvec( $z$ )`

Roots and Factorization (Finite fields)

factor  $f$  mod  $p$ , roots `factormod( $f, p$ ), polrootsmod`  
factor  $f$  over  $\mathbf{F}_p[x]/(T)$ , roots `factormod( $f, [T, p]$ ), polrootsmod`  
squarefree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodSQF( $f, \{D\}$ )`  
distinct degree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodDDF( $f, \{D\}$ )`  
**Roots and Factorization ( $p$ -adic fields)**  
factor  $f$  over  $\mathbf{Q}_p$ , roots `factorpadic( $f, p, r$ ), polrootspadic`  
 $p$ -adic root of  $f$  congruent to  $a$  mod  $p$  `padicappr( $f, a$ )`  
Newton polygon of  $f$  for prime  $p$  `newtonpoly( $f, p$ )`  
Hensel lift  $A/\mathrm{lc}(A) = \prod_i B[i] \bmod p^e$  `polhensellift( $A, B, p, e$ )`  
 $T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i)) \in \mathbf{Z}_p[x]$  `polteichmuller( $T, p, e$ )`  
extensions of  $\mathbf{Q}_p$  of degree  $N$  `padicfields( $p, N$ )`

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of  $f$  up to  $n$  `polsym( $f, n$ )`  
Graeffe transform of  $f$ ,  $g(x^2) = f(x)f(-x)$  `polgraeffe( $f$ )`  
factor  $f$  over coefficient field `factor( $f$ )`  
cyclotomic factors of  $f \in \mathbf{Q}[X]$  `polcyclofactors( $f$ )`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).  
find irreducible  $T \in \mathbf{F}_p[x]$ ,  $\deg T = n$  `ffinit( $p, n, \{x\}$ )`  
Create  $t$  in  $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$  `t = ffggen( $T, 't$ )`  
... indirectly, with implicit  $T$  `t = ffggen( $q, 't$ ); T = t.mod`  
map  $m$  from  $\mathbf{F}_q \ni a$  to  $\mathbf{F}_{q^k} \ni b$  `m = ffembed( $a, b$ )`  
build  $K = \mathbf{F}_q[x]/(P)$  extending  $\mathbf{F}_q \ni a$ , `ffextend( $a, P$ )`  
evaluate map  $m$  on  $x$  `ffmap( $m, x$ )`  
inverse map of  $m$  `ffinvmap( $m$ )`  
compose maps  $m \circ n$  `ffcompomap( $m, n$ )`  
 $x$  as polmod over codomain of map  $m$  `ffmaprel( $m, x$ )`  
 $F^n$  over  $\mathbf{F}_q \ni a$  `fffrobenius( $a, n$ )`  
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$  `ffnbirred( $q, n$ )`

Formal &  $p$ -adic Series

truncate power series or  $p$ -adic number `truncate( $x$ )`  
valuation of  $x$  at  $p$  `valuation( $x, p$ )`  
**Dirichlet and Power Series**  
Taylor expansion around 0 of  $f$  w.r.t.  $x$  `taylor( $f, x$ )`  
Laurent series of closure  $F$  up to  $x^k$  `laurentseries( $f, k$ )`  
 $\sum a_k b_k t^k$  from  $\sum a_k t^k$  and  $\sum b_k t^k$  `serconvol( $a, b$ )`  
 $f = \sum a_k t^k$  from  $\sum (a_k/k!) t^k$  `serlaplace( $f$ )`  
reverse power series  $F$  so  $F(f(x)) = x$  `serreverse( $f$ )`  
remove terms of degree  $< n$  in  $f$  `serchop( $f, n$ )`  
Dirichlet series multiplication / division `dirmul, dirdiv( $x, y$ )`  
Dirichlet Euler product ( $b$  terms) `direuler( $p = a, b, expr$ )`

Transcendental and  $p$ -adic Functions

real, imaginary part of  $x$  `real( $x$ ), imag( $x$ )`  
absolute value, argument of  $x$  `abs( $x$ ), arg( $x$ )`  
square/ $n$ th root of  $x$  `sqrt( $x$ ), sqrtn( $x, n, \{&z\}$ )`  
all  $n$ -th roots of 1 `rootsof1( $n$ )`  
FFT of  $[f_0, \dots, f_{n-1}]$  `w = fftinit( $n$ ), fft/fftinw( $w, f$ )`  
trig functions `sin, cos, tan, cotan, sinc`  
inverse trig functions `asin, acos, atan`  
hyperbolic functions `sinh, cosh, tanh, cotanh`  
inverse hyperbolic functions `asinh, acosh, atanh`  
 $\log(x)$ ,  $\log(1+x)$ ,  $e^x$ ,  $e^x - 1$  `log, loglp, exp, expm1`  
Euler  $\Gamma$  function,  $\log \Gamma$ ,  $\Gamma'/\Gamma$  `gamma, lngamma, psi`  
half-integer gamma function  $\Gamma(n+1/2)$  `gammah( $n$ )`  
Riemann's zeta  $\zeta(s) = \sum n^{-s}$  `zeta( $s$ )`  
 $\sum_{n \leq N} n^s$  `dirpowerssum( $N, s$ )`  
Hurwitz's  $\zeta(s, x) = \sum (n+x)^{-s}$  `zetahurwitz( $s, x$ )`  
multiple zeta value (MZV),  $\zeta(s_1, \dots, s_k)$  `zetamult( $s, \{T\}$ )`  
all MZVs for weight  $\sum s_i = n$  `zetamultall( $n$ )`  
convert MZV id to  $[s_1, \dots, s_k]$  `zetamultconvert( $f, \{flag\}$ )`  
MZV dual sequence `zetamultdual( $s$ )`  
multiple polylog  $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$  `polylogmult( $s, z$ )`

incomplete  $\Gamma$  function ( $y = \Gamma(s)$ ) `incgam( $s, x, \{y\}$ )`  
complementary incomplete  $\Gamma$  `incgamc( $s, x$ )`  
 $\int_x^\infty e^{-t} dt/t$ ,  $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$  `eint1, erfc`  
elliptic integral of 1st and 2nd kind `ellK( $k$ ), ellE( $k$ )`  
dilogarithm of  $x$  `dilog( $x$ )`  
 $m$ -th polylogarithm of  $x$  `polylog( $m, x, \{flag\}$ )`  
 $U$ -confluent hypergeometric function `hyperu( $a, b, u$ )`  
Hypergeometric  ${}_pF_q(A, B; z)$  `hypergeom( $A, B, z$ )`  
Bessel  $J_n(x)$ ,  $J_{n+1/2}(x)$  `besselj( $n, x$ ), besseljh( $n, x$ )`  
Bessel  $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$  `(bessel)i, k, h1, h2, y`  
Airy functions  $A_i(x)$ ,  $B_i(x)$  `airy( $x$ )`  
Lambert  $W$ :  $x$  s.t.  $xe^x = y$  `lambertw( $y$ )`  
Teichmuller character of  $p$ -adic  $x$  `teichmuller( $x$ )`

Iterations, Sums & Products

**Numerical integration for meromorphic functions**  
Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or  $\pm\infty$  (decreasing at least as  $x^{-2}$ ) or  $(x-a)^{-\alpha}$  singularity `[a, a]`  
exponential decrease  $e^{-\alpha|x|}$  `[ $\pm\infty, a$ ],  $\alpha > 0$`   
slow decrease  $|x|^\alpha$  `...  $\alpha < -1$`   
oscillating as  $\cos(kx)$   `$\alpha = kI$ ,  $k > 0$`   
oscillating as  $\sin(kx)$   `$\alpha = -kI$ ,  $k > 0$`   
numerical integration `intnum( $x = a, b, f, \{T\}$ )`  
weights  $T$  for `intnum` `intnuminit( $a, b, \{m\}$ )`  
weights  $T$  incl. kernel  $K$  `intfuncinit( $t = a, b, K, \{m\}$ )`  
integrate  $(2i\pi)^{-1} f$  on circle  $|z-a|=R$  `intcirc( $x = a, R, f, \{T\}$ )`

Other integration methods

$n$ -point Gauss-Legendre `intnumgauss( $x = a, b, f, \{n\}$ )`  
weights for  $n$ -point Gauss-Legendre `intnumgaussinit( $\{n\}$ )`  
Romberg integration (low accuracy) `intnumromb( $x = a, b, f, \{flag\}$ )`

Numerical summation

sum of series  $f(n)$ ,  $n \geq a$  (low accuracy) `suminf( $n = a, expr$ )`  
sum of alternating/positive series `sumalt, sumpos`  
sum of series using Euler-Maclaurin `sumnum( $n = a, f, \{T\}$ )`  
 $\sum_{n \geq a} F(n)$ ,  $F$  rational function `sumnumrat( $F, a$ )`  
 $\dots \sum_{p \geq a} F(p^s)$  `sumeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`  
weights for `sumnum`,  $a$  as in DE `sumnuminit( $\{\infty, a\}$ )`  
sum of series by Monien summation `sumnummonien( $n = a, f, \{T\}$ )`  
weights for `sumnummonien` `sumnummonieninit( $\{\infty, a\}$ )`  
sum of series using Abel-Plana `sumnumap( $n = a, f, \{T\}$ )`  
weights for `sumnumap`,  $a$  as in DE `sumnumapinit( $\{\infty, a\}$ )`  
sum of series using Lagrange `sumnumlagrange( $n = a, f, \{T\}$ )`  
weights for `sumnumlagrange` `sumnumlagrangeinit`

Products

product  $a \leq X \leq b$ , initialized at  $x$  `prod( $X = a, b, expr, \{x\}$ )`  
product over primes  $a \leq X \leq b$  `prodeuler( $X = a, b, expr$ )`  
infinite product  $a \leq X \leq \infty$  `prodinf( $X = a, expr$ )`  
 $\prod_{n \geq a} F(n)$ ,  $F$  rational function `prodnunrat( $F, a$ )`  
 $\prod_{p \geq a} F(p^s)$  `prodeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`

Other numerical methods

real root of $f$ in $[a, b]$ ; bracketed root	<code>solve(<math>X = a, b, f</math>)</code>
...interval splitting, step $s$	<code>solvestep(<math>X = a, b, s, f, \{flag = 0\}</math>)</code>
limit of $f(t)$ , $t \rightarrow \infty$	<code>limitnum(<math>f, \{\alpha\}</math>)</code>
asymptotic expansion of $f$ (rational)	<code>asypnum(<math>f, \{\alpha\}</math>)</code>
... $N + 1$ terms as floats	<code>asypnumraw(<math>f, N, \{\alpha\}</math>)</code>
numerical derivation w.r.t $x$ : $f'(a)$	<code>derivnum(<math>x = a, f</math>)</code>
evaluate continued fraction $F$ at $t$	<code>contfraceval(<math>F, t, \{L\}</math>)</code>
power series to cont. fraction ( $L$ terms)	<code>contfracinit(<math>S, \{L\}</math>)</code>
Padé approximant (deg. denom. $\leq B$ )	<code>bestapprPade(<math>S, \{B\}</math>)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(<math>x</math>)</code>
bit number $n$ of integer $x$	<code>bittest(<math>x, n</math>)</code>
Hamming weight of integer $x$	<code>hammingweight(<math>x</math>)</code>
digits of integer $x$ in base $B$	<code>digits(<math>x, \{B = 10\}</math>)</code>
sum of digits of integer $x$ in base $B$	<code>sumdigits(<math>x, \{B = 10\}</math>)</code>
integer from digits	<code>fromdigits(<math>v, \{B = 10\}</math>)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round $x$ to nearest integer	<code>round(<math>x, \{\&amp;e\}</math>)</code>
truncate $x$	<code>truncate(<math>x, \{\&amp;e\}</math>)</code>
gcd/LCM of $x$ and $y$	<code>gcd(<math>x, y</math>), lcm(<math>x, y</math>)</code>
gcd of entries of a vector/matrix	<code>content(<math>x</math>)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in $v$ to prime table	<code>addprimes(<math>v</math>)</code>
remove primes from prime table	<code>removeprimes(<math>v</math>)</code>
Chebyshev $\pi(x)$ , $n$ -th prime $p_n$	<code>primepi(<math>x</math>), prime(<math>n</math>)</code>
vector of first $n$ primes	<code>primes(<math>n</math>)</code>
smallest prime $\geq x$	<code>nextprime(<math>x</math>)</code>
largest prime $\leq x$	<code>precprime(<math>x</math>)</code>
factorization of $x$	<code>factor(<math>x, \{lim\}</math>)</code>
...selecting specific algorithms	<code>factorint(<math>x, \{flag = 0\}</math>)</code>
$n = df^2$ , $d$ squarefree/fundamental	<code>core(<math>n, \{fl\}</math>), coredisc</code>
certificate for (prime) $N$	<code>primecert(<math>N</math>)</code>
verifies a certificate $c$	<code>primecertisvalid(<math>c</math>)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover $x$ from its factorization	<code>factorback(<math>f, \{e\}</math>)</code>
$x \in \mathbf{Z}$ , $ x  \leq X$ , $\gcd(N, P(x)) \geq N$	<code>zncoppersmith(<math>P, N, X, \{B\}</math>)</code>
divisors of $N$ in residue class $r$ mod $s$	<code>divisorslensstra(<math>N, r, s</math>)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(<math>n</math>), bigomega</code>
divisors of $n$ / number of divisors $\tau(n)$	<code>divisors(<math>n</math>), numdiv</code>
sum of ( $k$ -th powers of) divisors of $n$	<code>sigma(<math>n, \{k\}</math>)</code>
Möbius $\mu$ -function	<code>moebius(<math>x</math>)</code>
Ramanujan's $\tau$ -function	<code>ramanujantau(<math>x</math>)</code>

Combinatorics

factorial of $x$	<code>x!</code> or <code>factorial(<math>x</math>)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial(<math>x, \{k\}</math>)</code>
Bernoulli number $B_n$ as real/rational	<code>bernreal(<math>n</math>), bernfrac</code>
$[B_0, B_2, \dots B_{2k}]$	<code>bernvec(<math>k</math>)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(<math>n, \{x\}</math>)</code>
Euler numbers	<code>eulerfrac, eulervec</code>
Euler polynomials $E_n(x)$	<code>eulerpol(<math>n, \{x\}</math>)</code>
Eulerian polynomials $A_n(x)$	<code>eulerianpol</code>
Fibonacci number $F_n$	<code>fibonacci(<math>n</math>)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(<math>n, k, \{flag\}</math>)</code>

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number of partitions of $n$	<code>numbpart(<math>n</math>)</code>
$k$ -th permutation on $n$ letters	<code>numtoperm(<math>n, k</math>)</code>
...index $k$ of permutation $v$	<code>permtotnum(<math>v</math>)</code>
order of permutation $p$	<code>permorder(<math>p</math>)</code>
signature of permutation $p$	<code>permsign(<math>p</math>)</code>
cyclic decomposition of permutation $p$	<code>permcycles(<math>p</math>)</code>
<b>Multiplicative groups <math>(\mathbf{Z}/N\mathbf{Z})^*</math>, <math>\mathbf{F}_q^*</math></b>	
Euler $\phi$ -function	<code>eulerphi(<math>x</math>)</code>
multiplicative order of $x$ (divides $\phi$ )	<code>znorder(<math>x, \{o\}</math>), fford</code>
primitive root mod $q$ / $x$ .mod	<code>znprimroot(<math>q</math>), ffpriroot(<math>x</math>)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(<math>n</math>)</code>
discrete logarithm of $x$ in base $g$	<code>znlog(<math>x, g, \{o\}</math>), ffflog</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(<math>x, y</math>)</code>
quadratic Hilbert symbol (at $p$ )	<code>hilbert(<math>x, y, \{p\}</math>)</code>
<b>Euclidean algorithm, continued fractions</b>	
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(<math>x, y</math>)</code>
minimal $u, v$ so $xu + yv = \gcd(x, y)$	<code>gcdext(<math>x, y</math>)</code>
half-gcd algorithm	<code>halfgcd(<math>x, y</math>)</code>
continued fraction of $x$	<code>confrac(<math>x, \{b\}, \{lmax\}</math>)</code>
last convergent of continued fraction $x$	<code>confracpnqn(<math>x</math>)</code>
rational approximation to $x$ (den. $\leq B$ )	<code>bestappr(<math>x, \{B\}</math>)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(<math>x, T</math>)</code>
<b>Miscellaneous</b>	
integer square / $n$ -th root of $x$	<code>sqrtnint(<math>x, n</math>)</code>
largest integer $e$ s.t. $b^e \leq x$ , $e = \lfloor \log_b(x) \rfloor$	<code>logint(<math>x, b, \{\&amp;z\}</math>)</code>
<b>Characters</b>	
Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure $G$ affording a .cyc method; e.g. <code>znstar(<math>q, 1</math>)</code> for Dirichlet characters. A character $\chi$ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$ .	
$\chi \cdot \psi$ ; $\chi^{-1}$ ; $\chi \cdot \psi^{-1}$ ; $\chi^k$	<code>charmul, charconj, chardiv, charpow</code>
order of $\chi$	<code>charorder(<math>cyc, \chi</math>)</code>
kernel of $\chi$	<code>charker(<math>cyc, \chi</math>)</code>
$\chi(x)$ , $G$ a GP group structure	<code>chareval(<math>G, \chi, x, \{z\}</math>)</code>
Galois orbits of characters	<code>chargalois(<math>G</math>)</code>
<b>Dirichlet Characters</b>	
initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar(<math>q, 1</math>)</code>
convert datum $D$ to $[G, \chi]$	<code>znchar(<math>D</math>)</code>
is $\chi$ odd?	<code>zncharisodd(<math>G, \chi</math>)</code>
real $\chi \rightarrow$ Kronecker symbol $(D/\cdot)$	<code>znchartokronecker(<math>G, \chi</math>)</code>
conductor of $\chi$	<code>zncharconductor(<math>G, \chi</math>)</code>
$[G_0, \chi_0]$ primitive attached to $\chi$	<code>znchartoprimitive(<math>G, \chi</math>)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(<math>G, \chi, N</math>)</code>
$\chi p$	<code>znchardecompose(<math>G, \chi, p</math>)</code>
$\prod_p  (Q, N) \chi p$	<code>znchardecompose(<math>G, \chi, Q</math>)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(<math>G, \chi</math>)</code>
<b>Conrey labelling</b>	
Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(<math>G, m</math>)</code>
character $\rightarrow$ Conrey label	<code>znconreyexp(<math>G, \chi</math>)</code>
log on Conrey generators	<code>znconreylog(<math>G, m</math>)</code>
conductor of $\chi$ ( $\chi_0$ primitive)	<code>znconreyconductor(<math>G, \chi, \{\chi_0\}</math>)</code>

True-False Tests

is $x$ the disc. of a quadratic field?	<code>isfundamental(<math>x</math>)</code>
is $x$ a prime?	<code>isprime(<math>x</math>)</code>
is $x$ a strong pseudo-prime?	<code>ispseudoprime(<math>x</math>)</code>
is $x$ square-free?	<code>issquarefree(<math>x</math>)</code>
is $x$ a square?	<code>issquare(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a perfect power?	<code>ispower(<math>x, \{k\}, \{\&amp;n\}</math>)</code>
is $x$ a perfect power of a prime? ( $x = p^n$ )	<code>isprimepower(<math>x, \&amp;n</math>)</code>
... of a pseudoprime?	<code>ispseudoprimepower(<math>x, \&amp;n</math>)</code>
is $x$ powerful?	<code>ispowerful(<math>x</math>)</code>
is $x$ a totient? ( $x = \varphi(n)$ )	<code>istotient(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a polygonal number? ( $x = P(s, n)$ )	<code>ispolygonal(<math>x, s, \{\&amp;n\}</math>)</code>
is $pol$ irreducible?	<code>polisirreducible(<math>pol</math>)</code>

Graphic Functions

crude graph of $expr$ between $a$ and $b$	<code>plot(<math>X = a, b, expr</math>)</code>
<b>High-resolution plot</b> (immediate plot)	
plot $expr$ between $a$ and $b$	<code>ploto(<math>X = a, b, expr, \{flag\}, \{n\}</math>)</code>
plot points given by lists $lx, ly$	<code>plotthraw(<math>lx, ly, \{flag\}</math>)</code>
terminal dimensions	<code>plotsizes()</code>
<b>Rectwindow functions</b>	
init window $w$ , with size $x, y$	<code>plotinit(<math>w, x, y</math>)</code>
erase window $w$	<code>plotkill(<math>w</math>)</code>
copy $w$ to $w_2$ with offset $(dx, dy)$	<code>plotcopy(<math>w, w_2, dx, dy</math>)</code>
slice contents of $w$	<code>plotclip(<math>w</math>)</code>
scale coordinates in $w$	<code>plotscale(<math>w, x_1, x_2, y_1, y_2</math>)</code>
<code>ploto</code> in $w$	<code>plotrecth(<math>w, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
<code>plotthraw</code> in $w$	<code>plotrectthraw(<math>w, data, \{flag\}</math>)</code>
draw window $w_1$ at $(x_1, y_1), \dots$	<code>plotdraw(<math>[[w_1, x_1, y_1], \dots]</math>)</code>

Low-level Rectwindow Functions

set current drawing color in $w$ to $c$	<code>plotcolor(<math>w, c</math>)</code>
current position of cursor in $w$	<code>plotcursor(<math>w</math>)</code>
write $s$ at cursor's position	<code>plotstring(<math>w, s</math>)</code>
move cursor to $(x, y)$	<code>plotmove(<math>w, x, y</math>)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(<math>w, dx, dy</math>)</code>
draw a box to $(x_2, y_2)$	<code>plotbox(<math>w, x_2, y_2</math>)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(<math>w, dx, dy</math>)</code>
draw polygon	<code>plotlines(<math>w, lx, ly, \{flag\}</math>)</code>
draw points	<code>plotpoints(<math>w, lx, ly</math>)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(<math>w, dx, dy</math>)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(<math>w, dx, dy</math>)</code>

Convert to Postscript or Scalable Vector Graphics

The format $f$ is either "ps" or "svg".	
as <code>ploto</code>	<code>plotexport(<math>f, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
as <code>plotthraw</code>	<code>plotthrawexport(<math>f, lx, ly, \{flag\}</math>)</code>
as <code>plotdraw</code>	<code>plotexport(<math>f, [[w_1, x_1, y_1], \dots]</math>)</code>