

Algebraic Number Theory

(PARI-GP version 2.13.4)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb(a,b,c,{d})`
reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) `qfbred(x,{flag},{D},{l},{s})`
return $[y, g]$, $g \in \text{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced `qfbreds12(x)`
composition of forms $x*y$ or `qfbnucomp(x,y,l)`
 n -th power of form x^n or `qfbnupow(x,n)`
composition without reduction `qfbcomprow(x,y)`
 n -th power without reduction `qfbpowrow(x,n)`
prime form of disc. x above prime p `qfbprimeform(x,p)`
class number of disc. x `qfbclassno(x)`
Hurwitz class number of disc. x `qfbhclassno(x)`
solve $Q(x,y) = n$ in integers `qfbsolve(Q,n)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`
minimal polynomial of ω `quadpoly(x)`
discriminant of $\mathbf{Q}(\sqrt{x})$ `quaddisc(x)`
regulator of real quadratic field `quadregulator(x)`
fundamental unit in real $\mathbf{Q}(\sqrt{D})$ `quadunit(D,{w})`
class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit(D,{flag},{t})`
Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert(D,{flag})`
... using specific class invariant ($D < 0$) `polclass(D,{inv})`
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadray(D,f,{flag})`

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$. We denote $\theta = \bar{X}$ the canonical root of f in K . A nf structure contains a maximal order and allows operations on elements and ideals. A bnf adds class group and units. A bnr is attached to ray class groups and class field theory. A rnf is attached to relative extensions L/K .

init number field structure nf `nfinit(f,{flag})`
known integer basis B `nfinit([f,B])`
order maximal at $vp = [p_1, \dots, p_k]$ `nfinit([f,vp])`
order maximal at all $p \leq P$ `nfinit([f,P])`
certify maximal order `nfcertify(nf)`

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K $nf.pol$
number of real/complex places $nf.r1/r2/sign$
discriminant of nf $nf.disc$
primes ramified in nf $nf.p$
 T_2 matrix $nf.t2$
complex roots of F $nf.roots$
integral basis of \mathbf{Z}_K as powers of θ $nf.zk$
different/codifferent $nf.diff, nf.codiff$
index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$ $nf.index$
recompute nf using current precision `nfnewprec(nf)`
init relative rnf $L = K[Y]/(g)$ `rnfinit(nf,g)`
init bnf structure `bnfinit(f,l)`

bnf members:

same as nf , plus
underlying nf $bnf.nf$
class group, regulator $bnf.clgp, bnf.reg$
fundamental/torsion units $bnf.fu, bnf.tu$
add S -class group and units, yield $bnfS$ `bnfsunit(bnf,S)`

init class field structure bnr `bnrinit(bnf,m,{flag})`
bnr members: same as bnf , plus
underlying bnf $bnr.bnf$
big ideal structure $bnr.bid$
modulus m $bnr.mod$
structure of $(\mathbf{Z}_K/m)^*$ $bnr.zkst$

Fields, subfields, embeddings

Defining polynomials, embeddings
smallest poly defining $f = 0$ (slow) `polredabs(f,{flag})`
small poly defining $f = 0$ (fast) `polredbest(f,{flag})`
random Tschirnhausen transform of f `poltschirnhaus(f)`
 $\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic? `nfisincl(f,g), nfisisom`
reverse polmod $a = A(t) \bmod T(t)$ `modreverse(a)`
compositum of $\mathbf{Q}[t]/(f), \mathbf{Q}[t]/(g)$ `polcompositum(f,g,{flag})`
compositum of $K[t]/(f), K[t]/(g)$ `nfcompositum(nf,f,g,{flag})`
splitting field of K (degree divides d) `nfsplitting(nf,{d})`
signs of real embeddings of x `nfeltsign(nf,x,{pl})`
complex embeddings of x `nfeltembed(nf,x,{pl})`
 $T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$ `nfpolsturm(nf,T,{pl})`

Subfields, polynomial factorization

subfields (of degree d) of nf `nfsubfields(nf,{d})`
maximal subfields of nf `nfsubfieldsmax(nf)`
maximal CM subfield of nf `nfsubfieldscm(nf)`
 d -th degree subfield of $\mathbf{Q}(\zeta_n)$ `polsubcyclo(n,d,{v})`
roots of unity in nf `nfrootsof1(nf)`
roots of g belonging to nf `nfroots(nf,g)`
factor g in nf `nffactor(nf,g)`

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ `algdep(x,k)`
alg. dep. with pol. coeffs for series s `seralgdep(s,x,y)`
small linear rel. on coords of vector x `lindep(x)`

Basic Number Field Arithmetic (nf)

Number field elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis $nf.zk$).

Basic operations

$x + y$ `nfeltadd(nf,x,y)`
 $x \times y$ `nfeltmul(nf,x,y)`
 $x^n, n \in \mathbf{Z}$ `nfeltpow(nf,x,n)`
 x/y `nfeltdiv(nf,x,y)`
 $q = x \setminus y := \text{round}(x/y)$ `nfeltdivu(nf,x,y)`
 $r = x \% y := x - (x \setminus y)y$ `nfeltmod(nf,x,y)`
... $[q,r]$ as above `nfeltdivrem(nf,x,y)`
reduce x modulo ideal A `nfeltreduce(nf,x,A)`
absolute trace $\text{Tr}_{K/\mathbf{Q}}(x)$ `nfelttrace(nf,x)`
absolute norm $N_{K/\mathbf{Q}}(x)$ `nfeltnorm(nf,x)`

Multiplicative structure of K^* ; $K^*/(K^*)^n$

valuation $v_p(x)$ `nfeltval(nf,x,p)`
... write $x = \pi^{v_p(x)}y$ `nfeltval(nf,x,p,&y)`
quadratic Hilbert symbol (at p) `nfhilbert(nf,a,b,{p})`
 b such that $xb^n = v$ is small `idealredmodpower(nf,x,n)`

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$ `nfbasis(f)`
field discriminant of $\mathbf{Q}[x]/(f)$ `nfdisc(f)`
... and factorization `nfdiscfactors(f)`
express x on integer basis `nfalgtobasis(nf,x)`
express element x as a polmod `nfbasistoalg(nf,x)`

Dedekind Zeta Function ζ_K , Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$ encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) < b$ `dirzetak(nf,b)`

init $\zeta_K^{(k)}(s)$ for $k \leq n$ `L = lfunitinit(bnf,R,{n=0})`
compute $\zeta_K(s)$ (n -th derivative) `lfun(L,s,{n=0})`
compute $\Lambda_K(s)$ (n -th derivative) `lfunlambd(L,s,{n=0})`

init $L_K^{(k)}(s, \chi)$ for $k \leq n$ `L = lfunitinit([bnr,chi],R,{n=0})`
compute $L_K(s, \chi)$ (n -th derivative) `lfun(L,s,{n})`
Artin root number of K `bnrrootnumber(bnr,chi,{flag})`
 $L(1, \chi)$, for all χ trivial on H `bnrL1(bnr,{H},{flag})`

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually bnr (ray class field), bnr, H (congruence subgroup) or bnr, χ (character on `bnr.clgp`). Any of these define a unique abelian extension of K .

units / S -units `bnfunits(bnf,{S})`
remove GRH assumption from bnf `bnfcertify(bnf)`
expo. of ideal x on class gp `bnfisprincipal(bnf,x,{flag})`
expo. of ideal x on ray class gp `bnrisprincipal(bnr,x,{flag})`
expo. of x on fund. units `bnfisunit(bnf,x)`
... on S -units, U is `bnfunits(bnf,S)` `bnfisunit(bnfs,U)`
signs of real embeddings of $bnf.fu$ `bnfsignunit(bnf)`
narrow class group `bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m `bnrclassno(bnf,m)`
discriminant of class field `bnrdisc(a1,{a2})`
ray class numbers, l list of moduli `bnrclassnolist(bnf,l)`
discriminants of class fields `bnrdisclist(bnf,l,{arch},{flag})`
decode output from `bnrdisclist` `bnfdecodemodule(nf,fa)`
is modulus the conductor? `bnrisconductor(a1,{a2})`
is class field (bnr,H) Galois over K^G `bnrisgalois(bnr,G,H)`
action of automorphism on `bnr.gen` `bnrgaloismatrix(bnr,aut)`
apply `bnrgaloismatrix` M to H `bnrgaloisapply(bnr,M,H)`
characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$ `bnrchar(bnr,g,{v})`
conductor of character χ `bnrconductor(bnr,chi)`
conductor of extension `bnrconductor(a1,{a2},{flag})`
conductor of extension $K[Y]/(g)$ `rnfconductor(bnf,g)`
canonical projection $\text{Cl}_F \rightarrow \text{Cl}_f, f \mid F$ `bnrmmap`
Artin group of extension $K[Y]/(g)$ `rnfnormgroup(bnr,g)`
subgroups of bnr , index $\leq b$ `subgrouplist(bnr,b,{flag})`
class field defined by $H \subset \text{Cl}_f$ `bnrclassfield(bnr,H)`
... low level equivalent, prime degree `rnfkummer(bnr,H)`
same, using Stark units (real field) `bnrstark(bnr,sub,{flag})`
is a an n -th power in K_v ? `nfislocalpower(nf,v,a,n)`
cyclic L/K satisf. local conditions `nfgrunwaldwang(nf,P,D,pl)`

Logarithmic class group

logarithmic ℓ -class group	<code>bnflog(<i>bnf</i>, ℓ)</code>
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	<code>bnflogcoef(<i>bnf</i>, <i>pr</i>)</code>
$\exp \deg_F(A)$	<code>bnflogdegree(<i>bnf</i>, <i>A</i>, ℓ)</code>
is ℓ -extension L/K locally cyclotomic	<code>rnfislocalcyclo(<i>rnf</i>)</code>

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	<code>nfisideal(<i>nf</i>, <i>id</i>)</code>
is x principal in bnf ?	<code>bnfisprincipal(<i>bnf</i>, <i>x</i>)</code>
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	<code>idealtwoelt(<i>nf</i>, <i>x</i>, {<i>a</i>})</code>
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	<code>idealhnf(<i>nf</i>, <i>a</i>, {<i>b</i>})</code>
norm of ideal x	<code>idealnrm(<i>nf</i>, <i>x</i>)</code>
minimum of ideal x (direction v)	<code>idealmin(<i>nf</i>, <i>x</i>, <i>v</i>)</code>
LLL-reduce the ideal x (direction v)	<code>idealred(<i>nf</i>, <i>x</i>, {<i>v</i>})</code>

Ideal Operations

add ideals x and y	<code>idealadd(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
multiply ideals x and y	<code>idealmul(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
intersection of ideal x with Q	<code>idealdown(<i>nf</i>, <i>x</i>)</code>
intersection of ideals x and y	<code>idealintersect(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
n -th power of ideal x	<code>idealpow(<i>nf</i>, <i>x</i>, <i>n</i>, {<i>flag</i>})</code>
inverse of ideal x	<code>idealinv(<i>nf</i>, <i>x</i>)</code>
divide ideal x by y	<code>idealdiv(<i>nf</i>, <i>x</i>, <i>y</i>, {<i>flag</i>})</code>
Find $(a, b) \in x \times y$, $a + b = 1$	<code>idealaddtoone(<i>nf</i>, <i>x</i>, {<i>y</i>})</code>
coprime integral A, B such that $x = A/B$	<code>idealnumden(<i>nf</i>, <i>x</i>)</code>

Primes and Multiplicative Structure

check whether x is a maximal ideal	<code>idealismaximal(<i>nf</i>, <i>x</i>)</code>
factor ideal x in \mathbf{Z}_K	<code>idealfactor(<i>nf</i>, <i>x</i>)</code>
expand ideal factorization in K	<code>idealfactorback(<i>nf</i>, <i>f</i>, {<i>e</i>})</code>
is ideal A an n -th power ?	<code>idealispower(<i>nf</i>, <i>A</i>, <i>n</i>)</code>
expand elt factorization in K	<code>nffactorback(<i>nf</i>, <i>f</i>, {<i>e</i>})</code>
decomposition of prime p in \mathbf{Z}_K	<code>idealprimedec(<i>nf</i>, <i>p</i>)</code>
valuation of x at prime ideal pr	<code>idealval(<i>nf</i>, <i>x</i>, <i>pr</i>)</code>
weak approximation theorem in nf	<code>idealchinese(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
$a \in K$, s.t. $v_{\mathfrak{p}}(a) = v_{\mathfrak{p}}(x)$ if $v_{\mathfrak{p}}(x) \neq 0$	<code>idealappr(<i>nf</i>, <i>x</i>)</code>
$a \in K$ such that $(a \cdot x, y) = 1$	<code>idealcoprime(<i>nf</i>, <i>x</i>, <i>y</i>)</code>
give bid =structure of $(\mathbf{Z}_K/id)^*$	<code>idealstar(<i>nf</i>, <i>id</i>, {<i>flag</i>})</code>
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$	<code>idealprincipalunits(<i>nf</i>, <i>pr</i>, <i>k</i>)</code>
discrete log of x in $(\mathbf{Z}_K/bid)^*$	<code>ideallog(<i>nf</i>, <i>x</i>, <i>bid</i>)</code>
idealstar of all ideals of norm $\leq b$	<code>ideallist(<i>nf</i>, <i>b</i>, {<i>flag</i>})</code>
add Archimedean places	<code>ideallistarch(<i>nf</i>, <i>b</i>, {<i>ar</i>}, {<i>flag</i>})</code>
init modpr structure	<code>nfmodprinit(<i>nf</i>, <i>pr</i>, {<i>v</i>})</code>
project t to \mathbf{Z}_K/pr	<code>nfmodpr(<i>nf</i>, <i>t</i>, <i>modpr</i>)</code>
lift from \mathbf{Z}_K/pr	<code>nfmodprlift(<i>nf</i>, <i>t</i>, <i>modpr</i>)</code>

Galois theory over Q

conjugates of a root θ of nf	<code>nfgaloisconj(<i>nf</i>, {<i>flag</i>})</code>
apply Galois automorphism s to x	<code>nfgaloisapply(<i>nf</i>, <i>s</i>, <i>x</i>)</code>
Galois group of field $\mathbf{Q}[x]/(f)$	<code>polgalois(<i>f</i>)</code>
initializes a Galois group structure G	<code>galoisinit(<i>pol</i>, {<i>den</i>})</code>
character table of G	<code>galoischartable(<i>G</i>)</code>
conjugacy classes of G	<code>galoisconjugacyclasses(<i>G</i>)</code>
$\det(1 - \rho(g)T)$, χ character of ρ	<code>galoischarpoly(<i>G</i>, χ, {<i>o</i>})</code>
$\det(\rho(g))$, χ character of ρ	<code>galoischarpoly(<i>G</i>, χ, {<i>o</i>})</code>
action of p in nfgaloisconj form	<code>galoispermtpol(<i>G</i>, {<i>p</i>})</code>
identify as abstract group	<code>galoisidentify(<i>G</i>)</code>
export a group for GAP/MAGMA	<code>galoisexport(<i>G</i>, {<i>flag</i>})</code>
subgroups of the Galois group G	<code>galoissubgroups(<i>G</i>)</code>
is subgroup H normal?	<code>galoisnormal(<i>G</i>, <i>H</i>)</code>

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subfields from subgroups	<code>galoissubfields(<i>G</i>, {<i>flag</i>}, {<i>v</i>})</code>
fixed field	<code>galoisfixedfield(<i>G</i>, <i>perm</i>, {<i>flag</i>}, {<i>v</i>})</code>
Frobenius at maximal ideal P	<code>idealfrobenius(<i>nf</i>, <i>G</i>, <i>P</i>)</code>
ramification groups at P	<code>idealramgroups(<i>nf</i>, <i>G</i>, <i>P</i>)</code>
is G abelian?	<code>galoisisabelian(<i>G</i>, {<i>flag</i>})</code>
abelian number fields/ \mathbf{Q}	<code>galoissubcyclo(<i>N</i>, <i>H</i>, {<i>flag</i>}, {<i>v</i>})</code>

The galpol package

query the package: polynomial	<code>galoisgetpol(<i>a</i>, <i>b</i>, {<i>s</i>})</code>
... : permutation group	<code>galoisgetgroup(<i>a</i>, <i>b</i>)</code>
... : group description	<code>galoisgetname(<i>a</i>, <i>b</i>)</code>

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	<code>rnfequation(<i>nf</i>, <i>T</i>, {<i>flag</i>})</code>
is L/K abelian?	<code>rnfisabelian(<i>nf</i>, <i>T</i>)</code>
relative nfalgtobasis	<code>rnfalgtobasis(<i>rnf</i>, <i>x</i>)</code>
relative nfbasistoalg	<code>rnfbasistoalg(<i>rnf</i>, <i>x</i>)</code>
relative idealhnf	<code>rnfidealhnf(<i>rnf</i>, <i>x</i>)</code>
relative idealmul	<code>rnfidealmul(<i>rnf</i>, <i>x</i>, <i>y</i>)</code>
relative idealtwoelt	<code>rnfidealtwoelt(<i>rnf</i>, <i>x</i>)</code>

Lifts and Push-downs

absolute \rightarrow relative representation for x	<code>rnfeltabstorel(<i>rnf</i>, <i>x</i>)</code>
relative \rightarrow absolute representation for x	<code>rnfeltreltoabs(<i>rnf</i>, <i>x</i>)</code>
lift x to the relative field	<code>rnfeltup(<i>rnf</i>, <i>x</i>)</code>
push x down to the base field	<code>rnfeltdown(<i>rnf</i>, <i>x</i>)</code>
idem for x ideal: (rnfideal)reltoabs, abstorel, up, down	

Norms and Trace

relative norm of element $x \in L$	<code>rnfeltnorm(<i>rnf</i>, <i>x</i>)</code>
relative trace of element $x \in L$	<code>rnfelttrace(<i>rnf</i>, <i>x</i>)</code>
absolute norm of ideal x	<code>rnfidealnrmabs(<i>rnf</i>, <i>x</i>)</code>
relative norm of ideal x	<code>rnfidealnrmrel(<i>rnf</i>, <i>x</i>)</code>
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	<code>bnfisintnorm(<i>bnf</i>, <i>x</i>)</code>
is $x \in \mathbf{Q}$ a norm from K ?	<code>bnfisnorm(<i>bnf</i>, <i>x</i>, {<i>flag</i>})</code>
initialize T for norm eq. solver	<code>rnfnorminit(<i>K</i>, <i>pol</i>, {<i>flag</i>})</code>
is $a \in K$ a norm from L ?	<code>rnfnisnorm(<i>T</i>, <i>a</i>, {<i>flag</i>})</code>
initialize t for Thue equation solver	<code>thueinit(<i>f</i>)</code>
solve Thue equation $f(x, y) = a$	<code>thue(<i>t</i>, <i>a</i>, {<i>sol</i>})</code>
characteristic poly. of a mod T	<code>rnfcharpoly(<i>nf</i>, <i>T</i>, <i>a</i>, {<i>v</i>})</code>

Factorization

factor ideal x in L	<code>rnfidealfactor(<i>rnf</i>, <i>x</i>)</code>
$[S, T]: T_{i,j} \mid S_i$; S primes of K above p	<code>rnfidealprimedec(<i>rnf</i>, <i>p</i>)</code>

Maximal order \mathbf{Z}_L as a \mathbf{Z}_K -module

relative polredbest	<code>rnfpolredbest(<i>nf</i>, <i>T</i>)</code>
relative polredabs	<code>rnfpolredabs(<i>nf</i>, <i>T</i>)</code>
relative Dedekind criterion, prime pr	<code>rnfdedekind(<i>nf</i>, <i>T</i>, <i>pr</i>)</code>
discriminant of relative extension	<code>rnfdisc(<i>nf</i>, <i>T</i>)</code>
pseudo-basis of \mathbf{Z}_L	<code>rnfpseudobasis(<i>nf</i>, <i>T</i>)</code>

General \mathbf{Z}_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$

relative HNF / SNF	<code>nfhnf(<i>nf</i>, <i>M</i>), nfsnf</code>
multiple of det M	<code>nfdetint(<i>nf</i>, <i>M</i>)</code>
HNF of M where $d = nfdetint(M)$	<code>nfhnfmod(<i>x</i>, <i>d</i>)</code>
reduced basis for M	<code>rnfilllgram(<i>nf</i>, <i>T</i>, <i>M</i>)</code>
determinant of pseudo-matrix M	<code>rnfdet(<i>nf</i>, <i>M</i>)</code>
Steinitz class of M	<code>rnfstinitz(<i>nf</i>, <i>M</i>)</code>

\mathbf{Z}_K -basis of M if \mathbf{Z}_K -free, or 0

n -basis of M , or $(n + 1)$ -generating set

is M a free \mathbf{Z}_K -module?

`rnfnhnbasis(bnf, M)`

`rnfbasis(bnf, M)`

`rnfisfree(bnf, M)`

Associative Algebras

A is a general associative algebra given by a multiplication table mt (over \mathbf{Q} or \mathbf{F}_p); represented by al from `algtableinit`.

create al from mt (over \mathbf{F}_p)	<code>algtableinit(<i>mt</i>, {<i>p</i> = 0})</code>
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	<code>algroup(<i>G</i>, {<i>p</i> = 0})</code>
center of group algebra	<code>algroupcenter(<i>G</i>, {<i>p</i> = 0})</code>

Properties

is (mt, p) OK for <code>algtableinit</code> ?	<code>algisassociative(<i>mt</i>, {<i>p</i> = 0})</code>
multiplication table mt	<code>algmtable(<i>al</i>)</code>
dimension of A over prime subfield	<code>algdim(<i>al</i>)</code>
characteristic of A	<code>algchar(<i>al</i>)</code>
is A commutative?	<code>algiscommutative(<i>al</i>)</code>
is A simple?	<code>algissimple(<i>al</i>)</code>
is A semi-simple?	<code>algissemisimple(<i>al</i>)</code>
center of A	<code>algcenter(<i>al</i>)</code>
Jacobson radical of A	<code>algradical(<i>al</i>)</code>
radical J and simple factors of A/J	<code>algsimpledec(<i>al</i>)</code>

Operations on algebras

create A/I , I two-sided ideal	<code>algquotient(<i>al</i>, <i>I</i>)</code>
create $A_1 \otimes A_2$	<code>algtensor(<i>al1</i>, <i>al2</i>)</code>
create subalgebra from basis B	<code>algsubalg(<i>al</i>, <i>B</i>)</code>
quotients by ortho. central idempotents e	<code>algcentralproj(<i>al</i>, <i>e</i>)</code>
isomorphic alg. with integral mult. table	<code>algmakeintegral(<i>mt</i>)</code>
prime subalgebra of semi-simple A over \mathbf{F}_p	<code>algprimesubalg(<i>al</i>)</code>
find isomorphism $A \cong M_d(\mathbf{F}_q)$	<code>algsplit(<i>al</i>)</code>

Operations on lattices in algebras

lattice generated by cols. of M	<code>alglathnf(<i>al</i>, <i>M</i>)</code>
... by the products xy , $x \in lat1$, $y \in lat2$	<code>alglatmul(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
sum $lat1 + lat2$ of the lattices	<code>alglatadd(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
intersection $lat1 \cap lat2$	<code>alglatinter(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
test $lat1 \subset lat2$	<code>alglatsubset(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
generalized index $(lat2 : lat1)$	<code>alglatindex(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
$\{x \in al \mid x \cdot lat1 \subset lat2\}$	<code>alglatlefttransporter(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
$\{x \in al \mid lat1 \cdot x \subset lat2\}$	<code>alglatrighttransporter(<i>al</i>, <i>lat1</i>, <i>lat2</i>)</code>
test $x \in lat$ (set c = coord. of x)	<code>alglatcontains(<i>al</i>, <i>lat</i>, <i>x</i>, {&<i>c</i>})</code>
element of lat with coordinates c	<code>alglatelement(<i>al</i>, <i>lat</i>, <i>c</i>)</code>

Operations on elements

$a + b$, $a - b$, $-a$	<code>algadd(<i>al</i>, <i>a</i>, <i>b</i>), algsub, algneg</code>
$a \times b$, a^2	<code>algmul(<i>al</i>, <i>a</i>, <i>b</i>), algsqr</code>
a^n , a^{-1}	<code>algpow(<i>al</i>, <i>a</i>, <i>n</i>), alginv</code>
is x invertible ? (then set $z = x^{-1}$)	<code>alginv(<i>al</i>, <i>x</i>, {&<i>z</i>})</code>
find z such that $x \times z = y$	<code>algdivl(<i>al</i>, <i>x</i>, <i>y</i>)</code>
find z such that $z \times x = y$	<code>algdivr(<i>al</i>, <i>x</i>, <i>y</i>)</code>
does s s.t. $x \times z = y$ exist? (set it)	<code>algisdivl(<i>al</i>, <i>x</i>, <i>y</i>, {&<i>z</i>})</code>
matrix of $v \mapsto x \cdot v$	<code>algtomatrix(<i>al</i>, <i>x</i>)</code>
absolute norm	<code>algnorm(<i>al</i>, <i>x</i>)</code>
absolute trace	<code>algtrace(<i>al</i>, <i>x</i>)</code>
absolute char. polynomial	<code>algcharpoly(<i>al</i>, <i>x</i>)</code>
given $a \in A$ and polynomial T , return $T(a)$	<code>algpoleval(<i>al</i>, <i>T</i>, <i>a</i>)</code>
random element in a box	<code>algrandom(<i>al</i>, <i>b</i>)</code>

Based on an earlier version by Joseph H. Silverman

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Central Simple Algebras

A is a central simple algebra over a number field K ; represented by al from **algnit**; K is given by a nf structure.
create CSA from data **algnit**($B, C, \{v\}, \{maxord = 1\}$)
multiplication table over K $B = K, C = mt$
cyclic algebra ($L/K, \sigma, b$) $B = rnf, C = [sigma, b]$
quaternion algebra $(a, b)_K$ $B = K, C = [a, b]$
matrix algebra $M_d(K)$ $B = K, C = d$
local Hasse invariants over K $B = K, C = [d, [PR, HF], HI]$

Properties

type of al (mt, CSA) **algtype**(al)
dimension of A over \mathbf{Q} **algdim**($al, 1$)
dimension of al over its center K **algdim**(al)
degree of A ($= \sqrt{\dim_K A}$) **algdegree**(al)
 al a cyclic algebra ($L/K, \sigma, b$); return σ **algaut**(al)
...return b **algb**(al)
...return L/K , as an rnf **algsplittingfield**(al)
split A over an extension of K **algsplittingdata**(al)
splitting field of A as an rnf over center **algsplittingfield**(al)
multiplication table over center **algrelmultable**(al)
places of K at which A ramifies **algramifiedplaces**(al)
Hasse invariants at finite places of K **alghassef**(al)
Hasse invariants at infinite places of K **alghassei**(al)
Hasse invariant at place v **alghasse**(al, v)
index of A over K (at place v) **algindex**($al, \{v\}$)
is al a division algebra? (at place v) **algisdivision**($al, \{v\}$)
is A ramified? (at place v) **algisramified**($al, \{v\}$)
is A split? (at place v) **algissplit**($al, \{v\}$)

Operations on elements

reduced norm **algnorm**(al, x)
reduced trace **algtrace**(al, x)
reduced char. polynomial **algcharpoly**(al, x)
express x on integral basis **algalgtobasis**(al, x)
convert x to algebraic form **algbasistoalg**(al, x)
map $x \in A$ to $M_d(L)$, L split. field **algtomatrix**(al, x)

Orders

Z-basis of order \mathcal{O}_0 **algbasis**(al)
discriminant of order \mathcal{O}_0 **algdisc**(al)
Z-basis of natural order in terms \mathcal{O}_0 's basis **alginvbasis**(al)